Quantum circuits w/ classical vs quantum control of causal orders

Cyril Branciard

Institut Néel – CNRS (Grenoble, France)

Joint work w/ Julian Wechs, Alastair Abbott, Hippolyte Dourdent

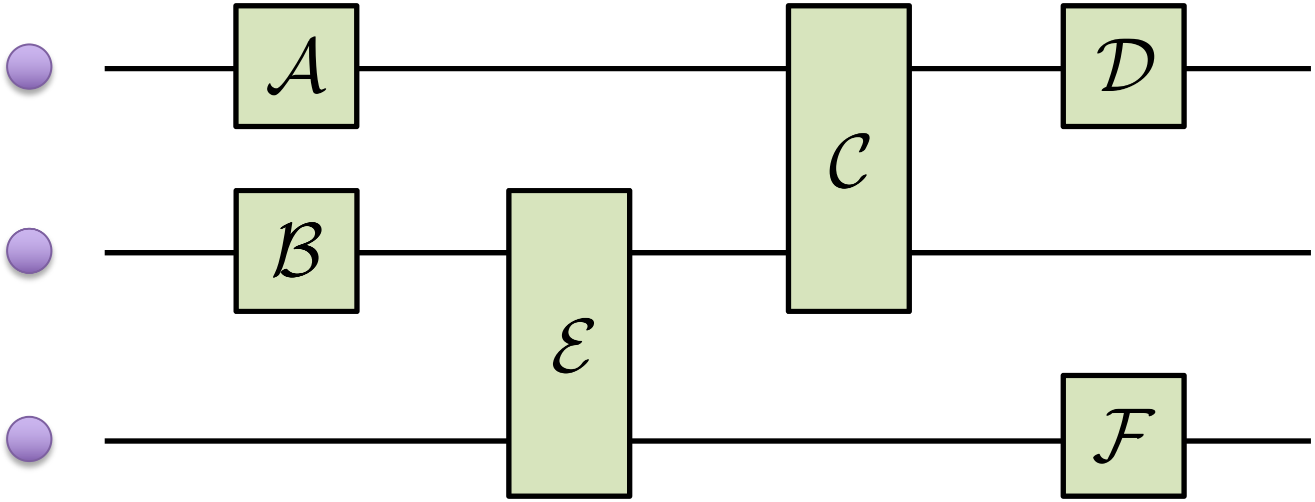
QISS Workshop — Hong Kong — Jan. 13-17, 2020

Quantum Engineering
Univ. Grenoble Alpes

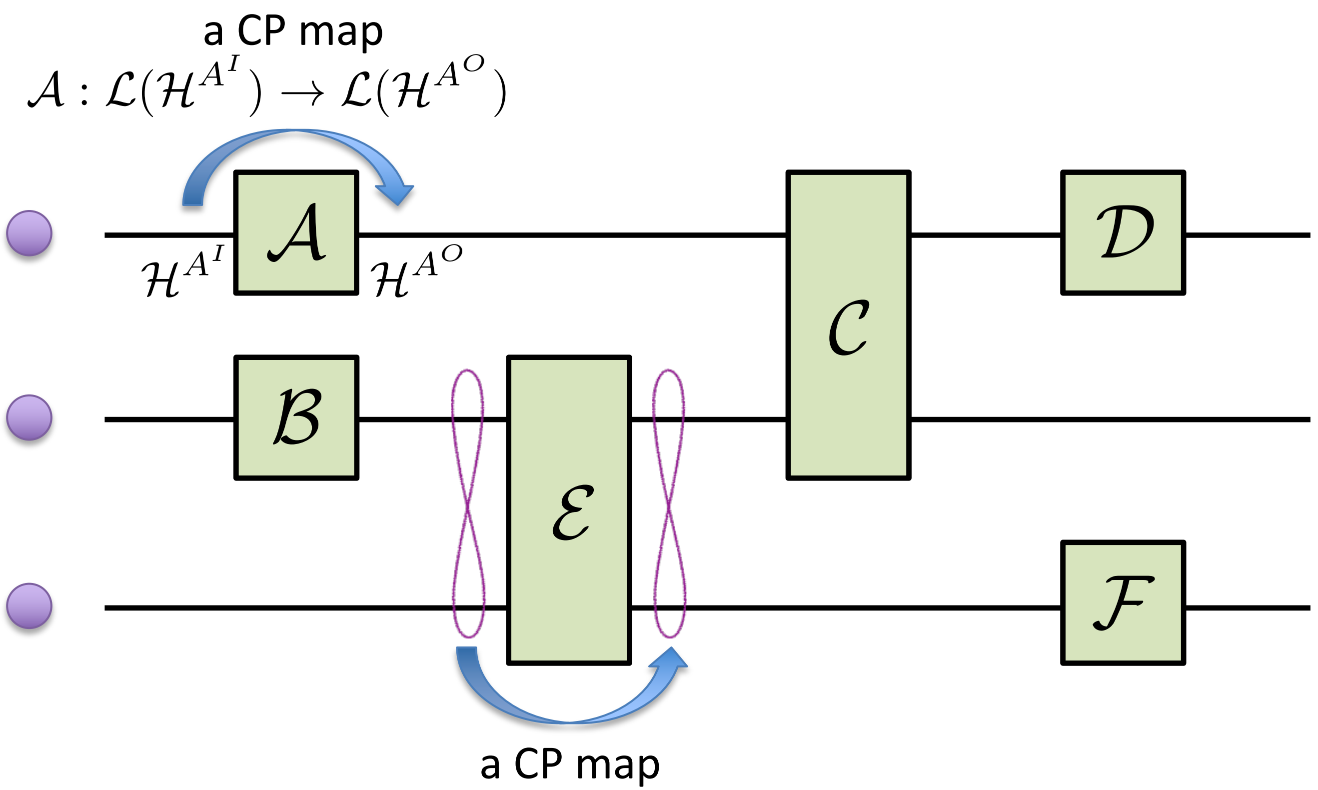
UGA
Univ. Grenoble Alpes

Agence Nationale de la Recherche
ANR

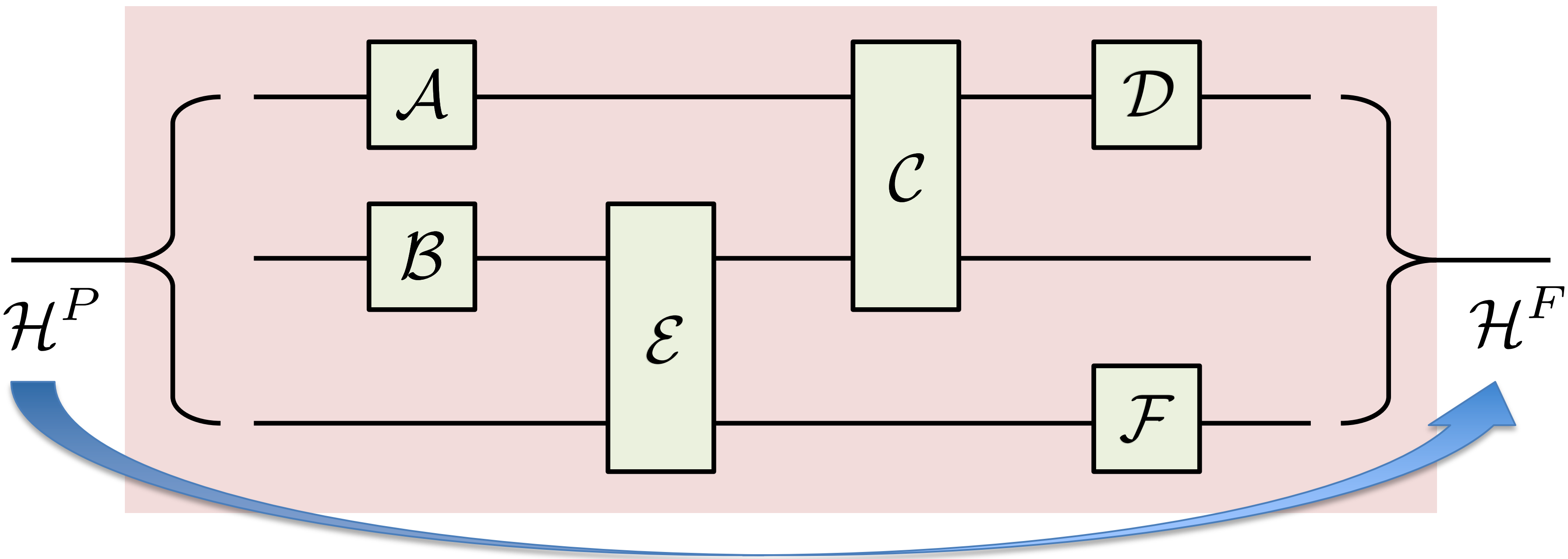
Quantum circuits



Quantum circuits



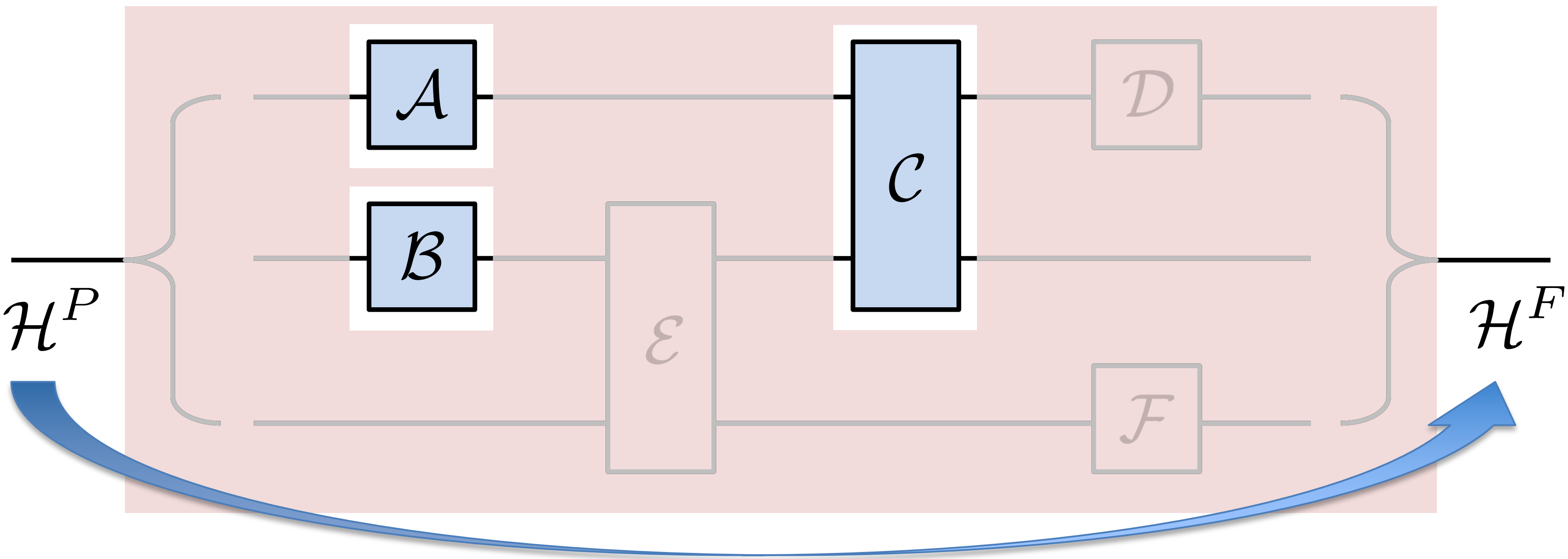
Quantum circuits as quantum supermaps



a CP map

$$\mathcal{M} : \mathcal{L}(\mathcal{H}^P) \rightarrow \mathcal{L}(\mathcal{H}^F)$$

Quantum circuits as quantum supermaps



a CP map

$$\mathcal{M} : \mathcal{L}(\mathcal{H}^P) \to \mathcal{L}(\mathcal{H}^F)$$

$\mathcal{M}(\mathcal{A}, \mathcal{B}, \mathcal{C})$ a quantum "supermap"

(that maps CP maps to a new CP map)

[Chiribella, D'Ariano & Perinotti, EPL 2008, PRL 2008, PRA 2009]

What kind of quantum circuits are possible?

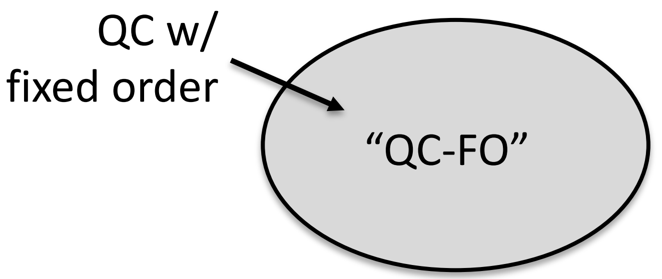
- Quantum Circuits with Fixed causal Order (of the CP maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$):

  "Quantum combs" [Chiribella, D'Ariano & Perinotti, EPL 2008, PRL 2008, PRA 2009]

What kind of quantum circuits are possible?

- Quantum Circuits with Fixed causal Order (of the CP maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$):

  "Quantum combs" [Chiribella, D'Ariano & Perinotti, EPL 2008, PRL 2008, PRA 2009]

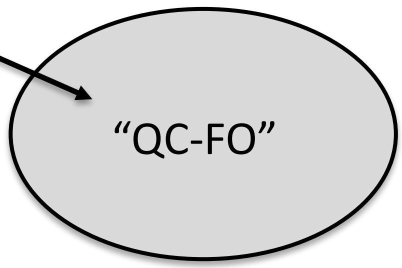


What kind of quantum circuits are possible?

- **Quantum Circuits with Fixed causal Order** (of the CP maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$):

  "Quantum combs" [Chiribella, D'Ariano & Perinotti, EPL 2008, PRL 2008, PRA 2009]

QC w/
fixed order



× "classical switch": $\begin{cases} c = 0 \rightarrow \mathcal{A} \text{ then } \mathcal{B} \\ c = 1 \rightarrow \mathcal{B} \text{ then } \mathcal{A} \end{cases}$

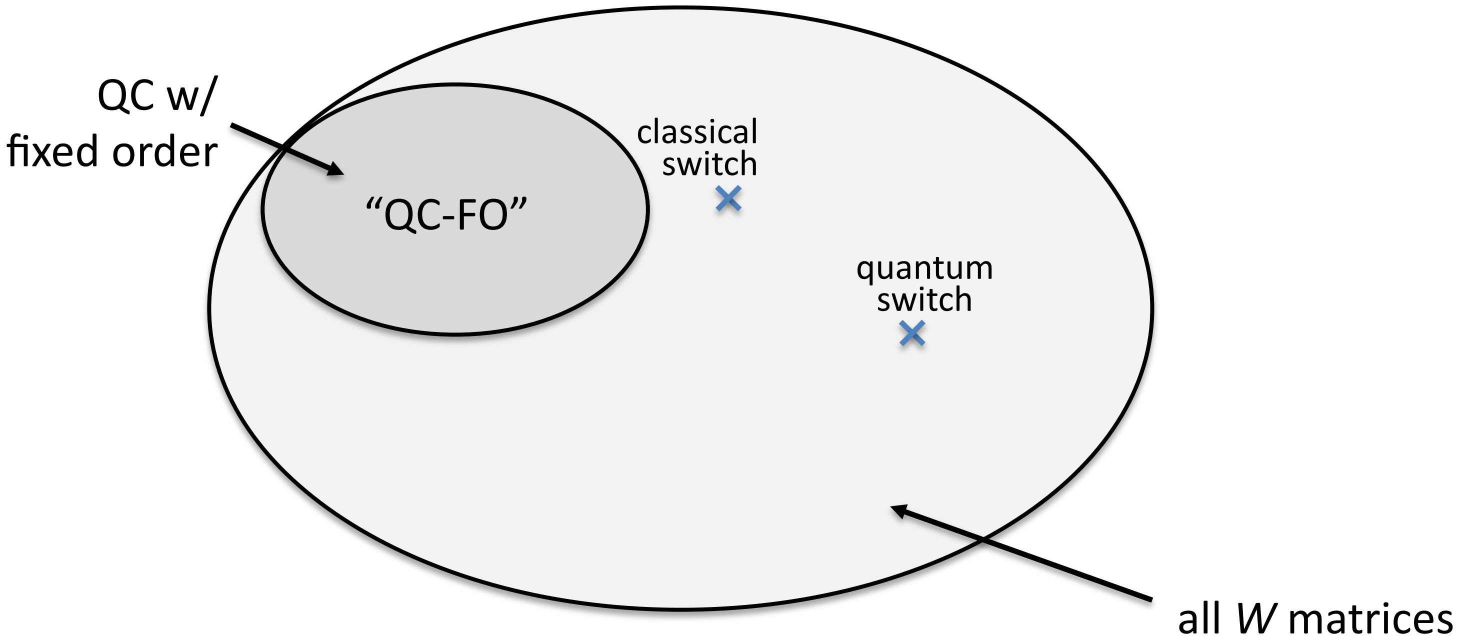
× "quantum switch": $\begin{cases} c = |0\rangle \rightarrow \mathcal{A} \text{ then } \mathcal{B} \\ c = |1\rangle \rightarrow \mathcal{B} \text{ then } \mathcal{A} \end{cases}$

[Chiribella *et al.*, arXiv 2009, PRA 2013]

What kind of quantum circuits are possible?

- Quantum Circuits with Fixed causal Order (of the CP maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$):

  "Quantum combs" [Chiribella, D'Ariano & Perinotti, EPL 2008, PRL 2008, PRA 2009]



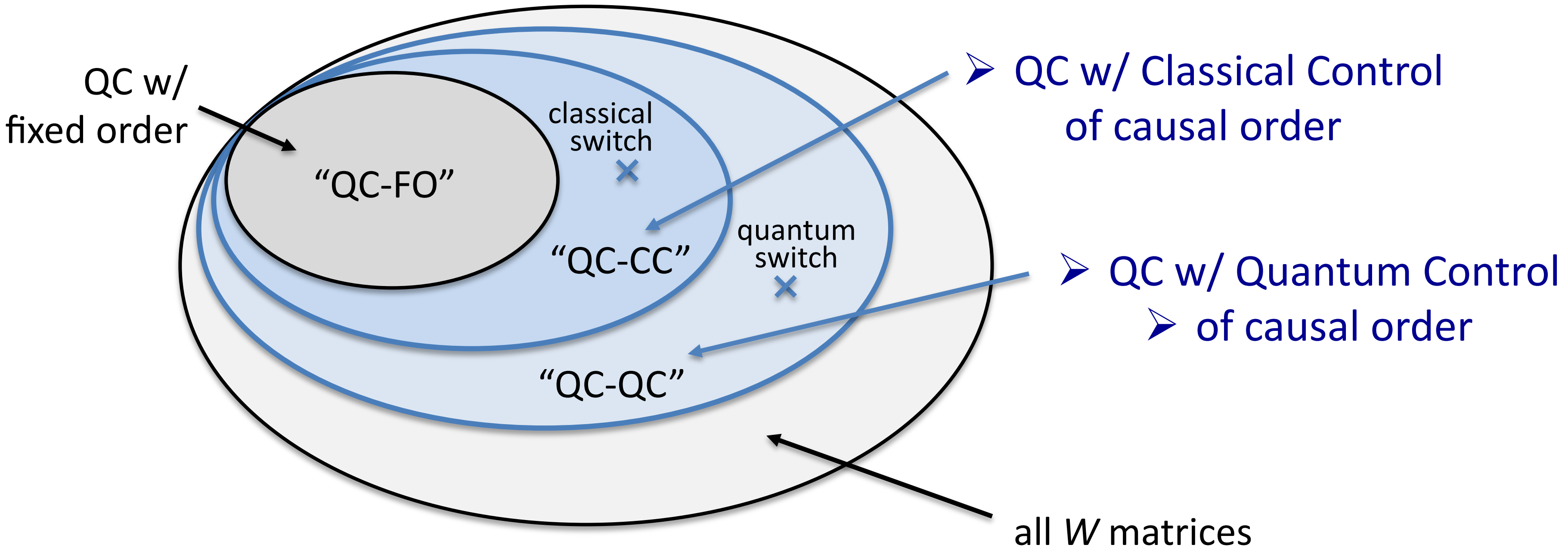
- Most general consistent quantum supermaps

  "Process matrices" W [Oreshkov, Costa & Brukner, Nat Comms 2012]

What kind of quantum circuits are possible?

- Quantum Circuits with Fixed causal Order (of the CP maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$):

  “Quantum combs” [Chiribella, D’Ariano & Perinotti, EPL 2008, PRL 2008, PRA 2009]



- Most general consistent quantum supermaps

  “Process matrices” W [Oreshkov, Costa & Brukner, Nat Comms 2012]

Outline

- Quantum Circuits with Fixed causal Order (QC-FOs)

- **Quantum Circuits with <u>Classical Control</u> of causal order** (QC-CCs)

- **Quantum Circuits with <u>Quantum Control</u> of causal order** (QC-QCs)

(Using the process matrix framework)

Outline

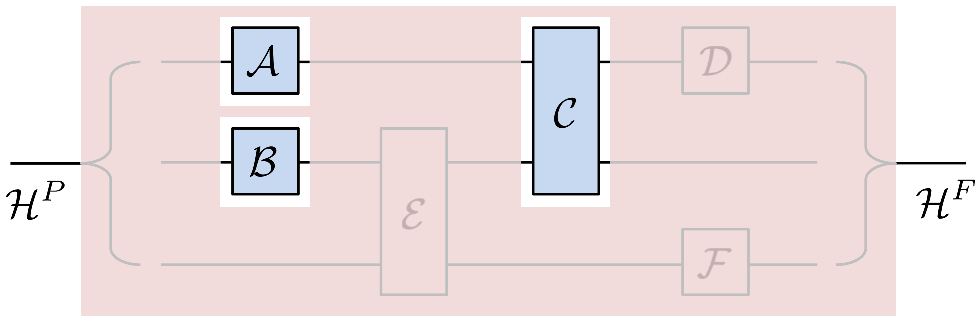
- Quantum Circuits with Fixed causal Order (QC-FOs)

- **Quantum Circuits with <u>Classical Control</u> of causal order** (QC-CCs)

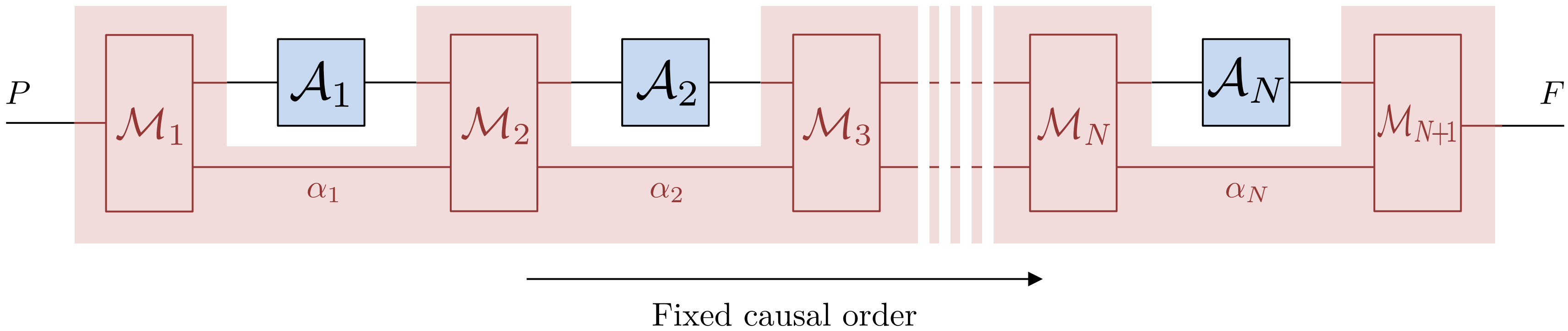
- **Quantum Circuits with <u>Quantum Control</u> of causal order** (QC-QCs)

(Using the process matrix framework)

Quantum Circuits with Fixed causal Order

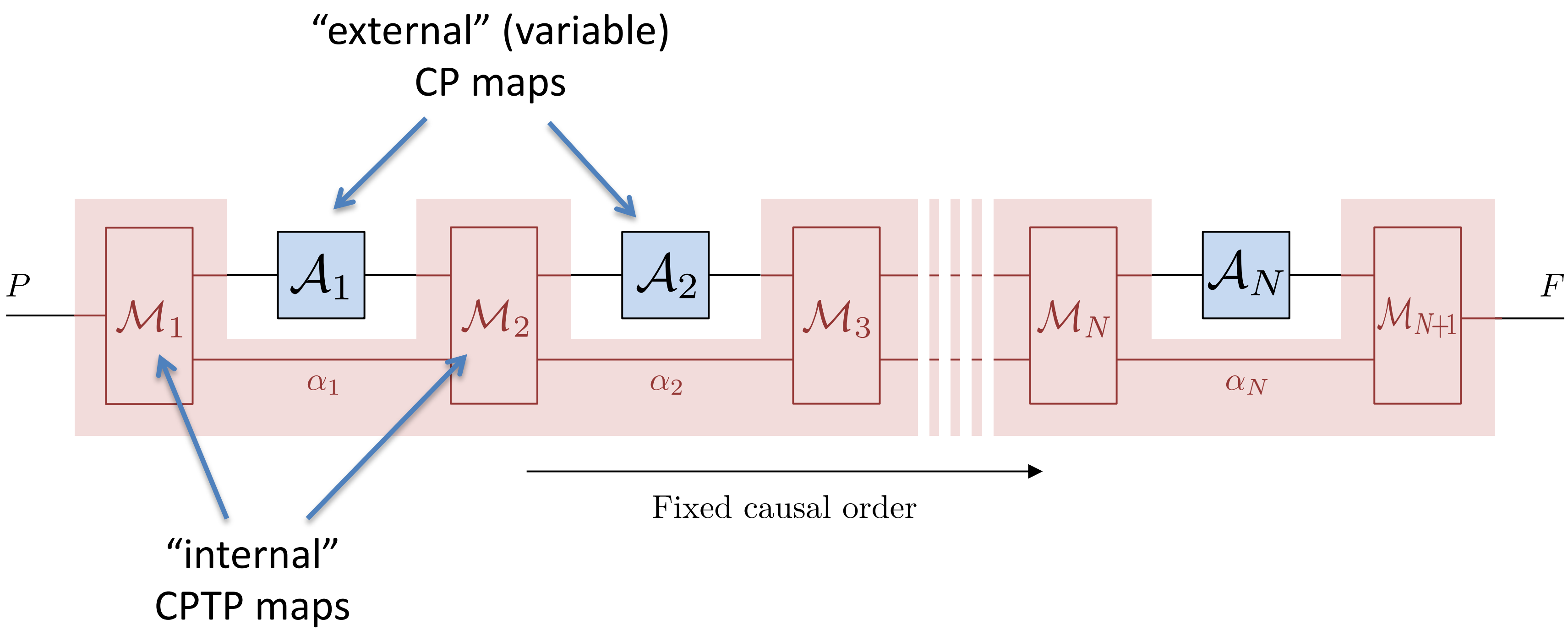


Quantum Circuits with Fixed causal Order



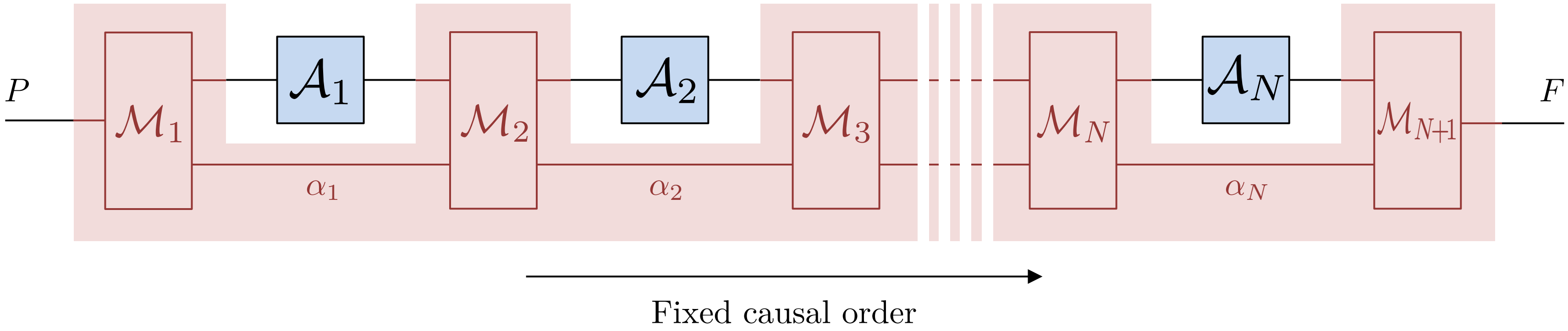
[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum Circuits with Fixed causal Order



[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum **C**ircuits with **F**ixed causal **O**rder

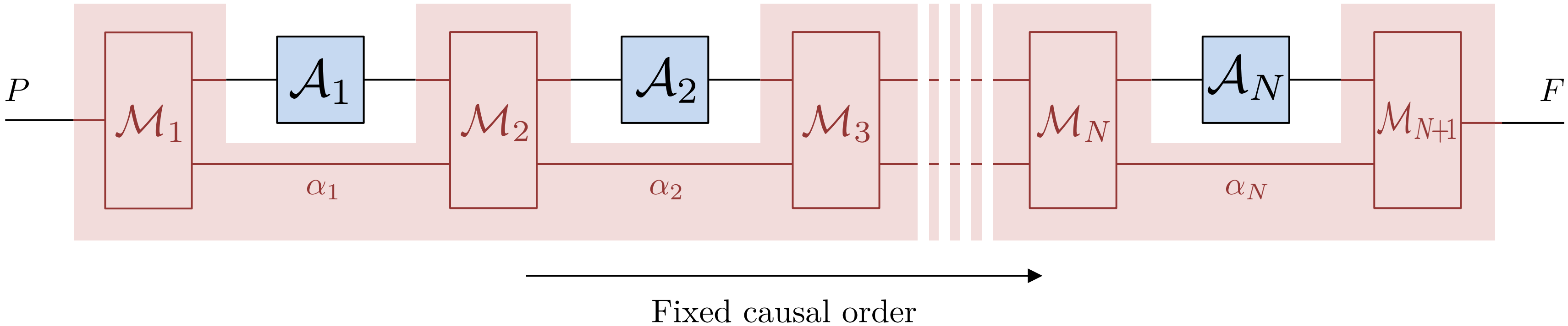


Global induced map:

$$\mathcal{M} = \mathcal{M}_{N+1} \circ \mathcal{A}_N \circ \mathcal{M}_N \circ \cdots \circ \mathcal{M}_3 \circ \mathcal{A}_2 \circ \mathcal{M}_2 \circ \mathcal{A}_1 \circ \mathcal{M}_1$$

[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum Circuits with Fixed causal Order



Global induced map:

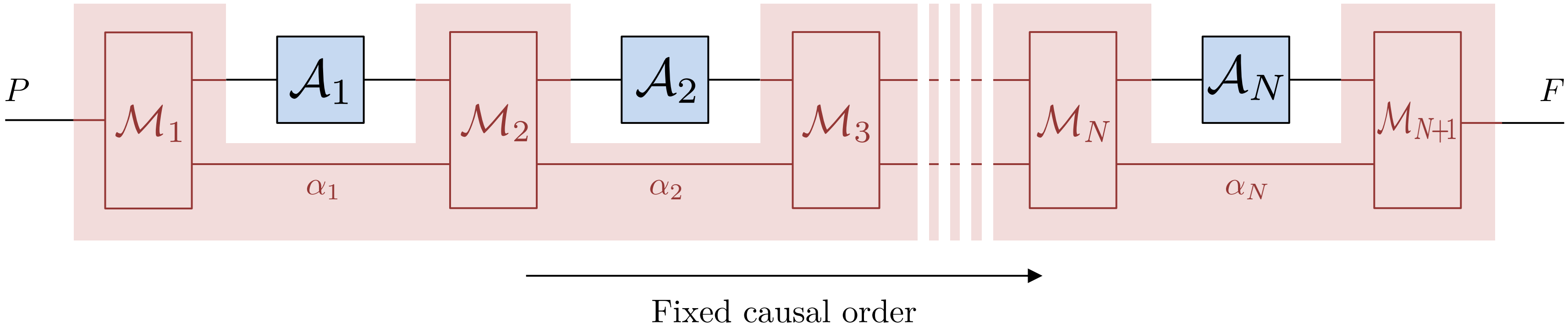
$$\mathcal{M} = \mathcal{M}_{N+1} \circ \mathcal{A}_N \circ \mathcal{M}_N \circ \cdots \circ \mathcal{M}_3 \circ \mathcal{A}_2 \circ \mathcal{M}_2 \circ \mathcal{A}_1 \circ \mathcal{M}_1$$

using Choi isomorphism:

$$\mathcal{M} : \mathcal{L}(\mathcal{H}^X) \to \mathcal{L}(\mathcal{H}^Y) \quad \Longleftrightarrow \quad M \in \mathcal{L}(\mathcal{H}^X) \otimes \mathcal{L}(\mathcal{H}^Y)$$

[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum Circuits with Fixed causal Order



Global induced map:

$$\mathcal{M} = \mathcal{M}_{N+1} \circ \mathcal{A}_N \circ \mathcal{M}_N \circ \cdots \circ \mathcal{M}_3 \circ \mathcal{A}_2 \circ \mathcal{M}_2 \circ \mathcal{A}_1 \circ \mathcal{M}_1$$

using Choi isomorphism:

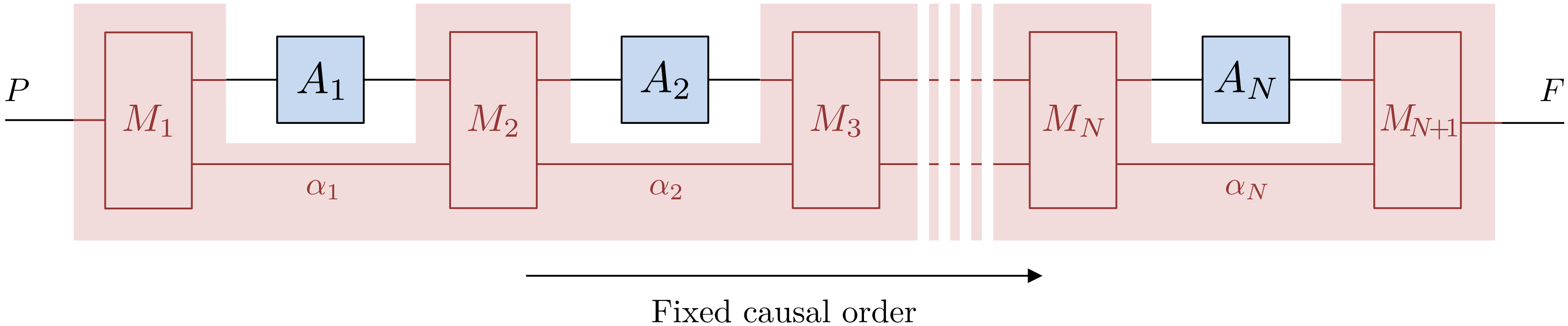
$$\mathcal{M} : \mathcal{L}(\mathcal{H}^X) \to \mathcal{L}(\mathcal{H}^Y) \iff M \in \mathcal{L}(\mathcal{H}^X) \otimes \mathcal{L}(\mathcal{H}^Y)$$

$$\mathcal{M} = \mathcal{M}_2 \circ \mathcal{M}_1 \iff M = M_2 * M_1$$

"link product"

[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum Circuits with Fixed causal Order



Global induced map:

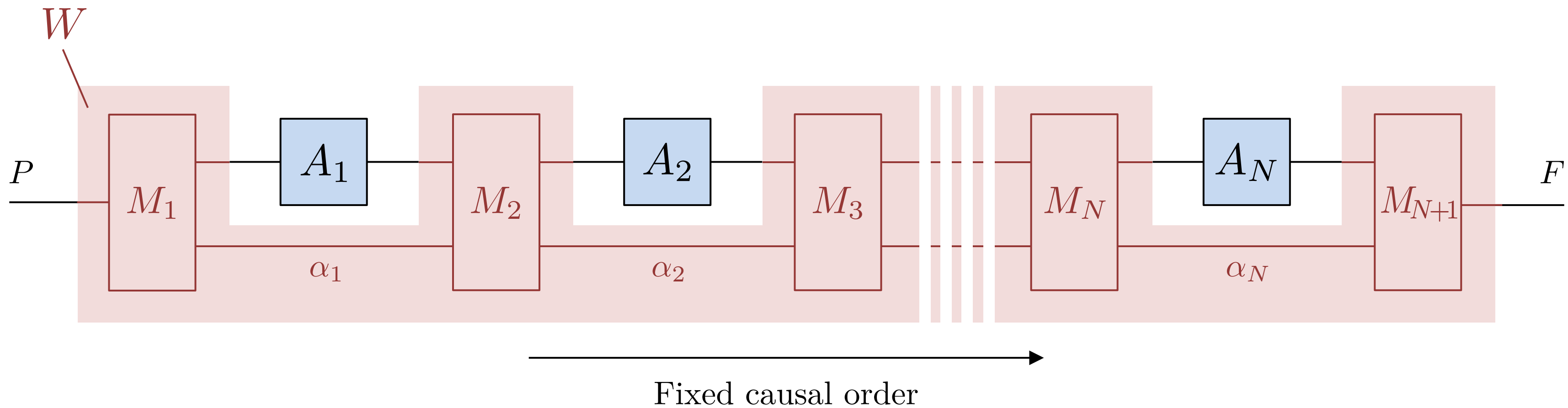
$$\mathcal{M} = \mathcal{M}_{N+1} \circ \mathcal{A}_N \circ \mathcal{M}_N \circ \cdots \circ \mathcal{M}_3 \circ \mathcal{A}_2 \circ \mathcal{M}_2 \circ \mathcal{A}_1 \circ \mathcal{M}_1$$

Choi

$$M = M_{N+1} * A_N * M_N * \cdots * M_3 * A_2 * M_2 * A_1 * M_1$$

[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum Circuits with Fixed causal Order



Global induced map:

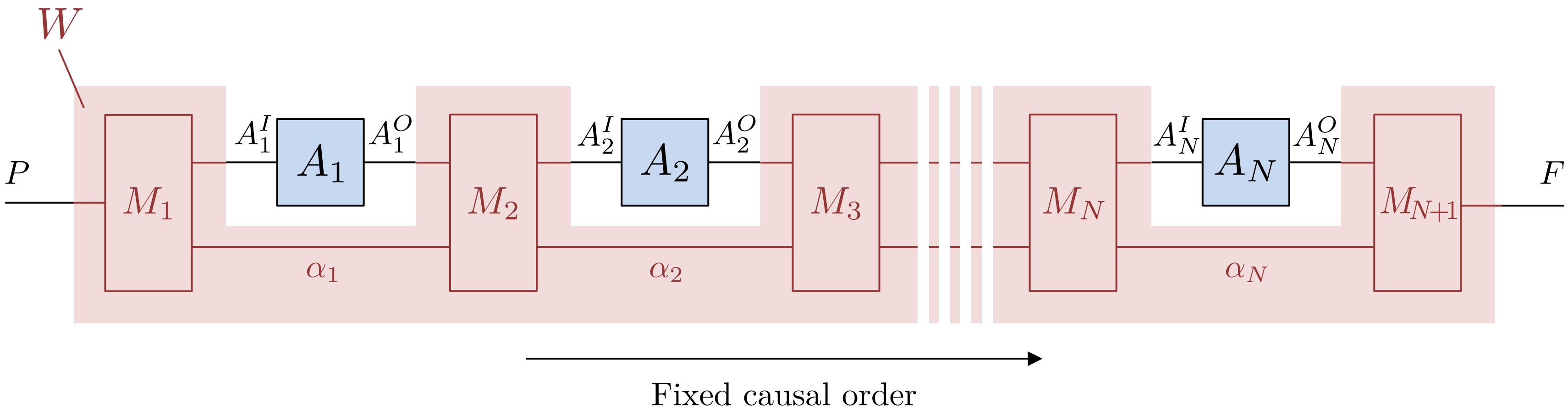
$$\mathcal{M} = \mathcal{M}_{N+1} \circ \mathcal{A}_N \circ \mathcal{M}_N \circ \cdots \circ \mathcal{M}_3 \circ \mathcal{A}_2 \circ \mathcal{M}_2 \circ \mathcal{A}_1 \circ \mathcal{M}_1$$

Choi

$$\begin{aligned} M &= M_{N+1} * A_N * M_N * \cdots * M_3 * A_2 * M_2 * A_1 * M_1 \\ &= \underbrace{\big(M_1 * M_2 * \cdots * M_N * M_{N+1}\big)}_{W} * \big(A_1 \otimes A_2 \otimes \cdots \otimes A_N\big) \end{aligned}$$

[Chiribella, D’Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum Circuits with Fixed causal Order



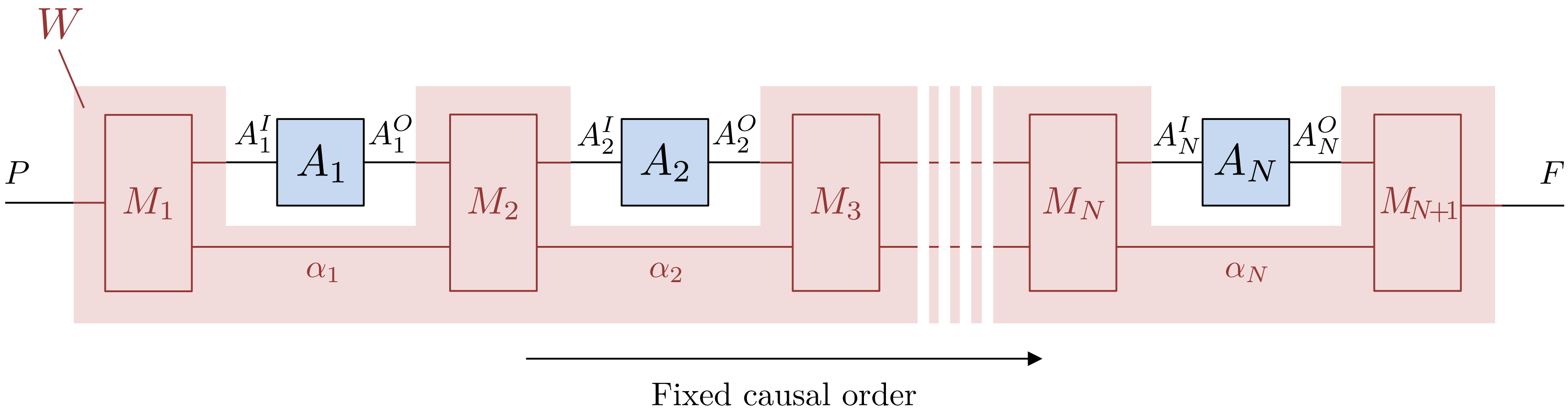
$$M = W * \left(A_1 \otimes A_2 \otimes \cdots \otimes A_N\right)$$

with $W = M_1 * M_2 * \cdots * M_N * M_{N+1}$

$W \in \mathcal{L}(\mathcal{H}^{P A_1^I A_1^O \cdots A_N^I A_N^O F})$: a "Quantum comb"

[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Quantum Circuits with Fixed causal Order



$$M = W * \left(A_1 \otimes A_2 \otimes \cdots \otimes A_N\right)$$

with $\quad W = M_1 * M_2 * \cdots * M_N * M_{N+1}$

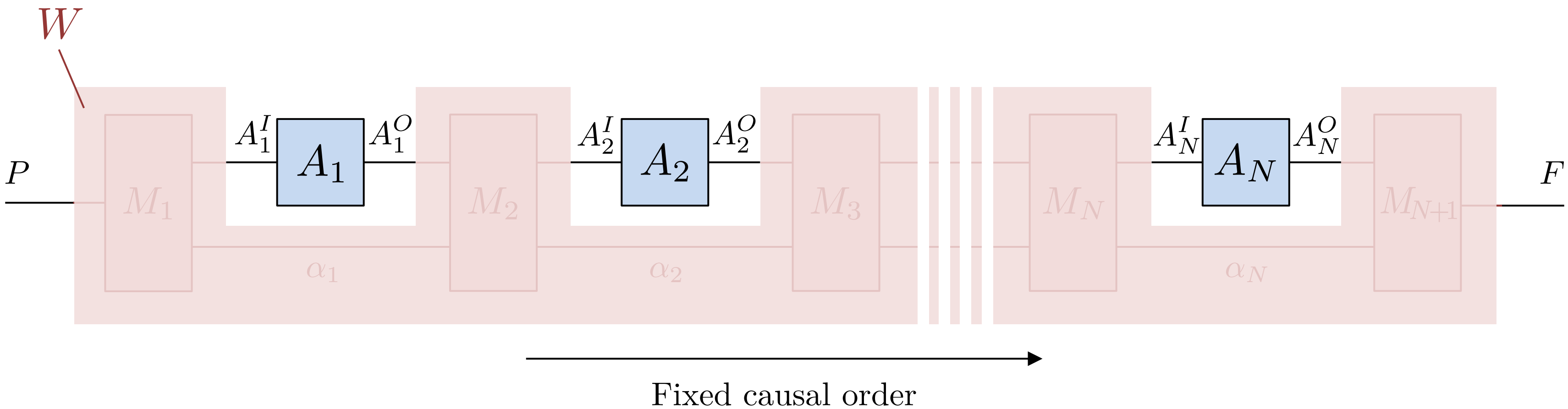
$W \in \mathcal{L}(\mathcal{H}^{P A_1^I A_1^O \cdots A_N^I A_N^O F})$: a "Quantum comb"

[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

(a "Process matrix" with fixed order)

[Oreshkov, Costa & Brukner, Nat Comms 2012]

Quantum Circuits with Fixed causal Order



$$M = W * \big(A_1 \otimes A_2 \otimes \cdots \otimes A_N\big)$$

with $\boxed{W} = M_1 * M_2 * \cdots * M_N * M_{N+1}$

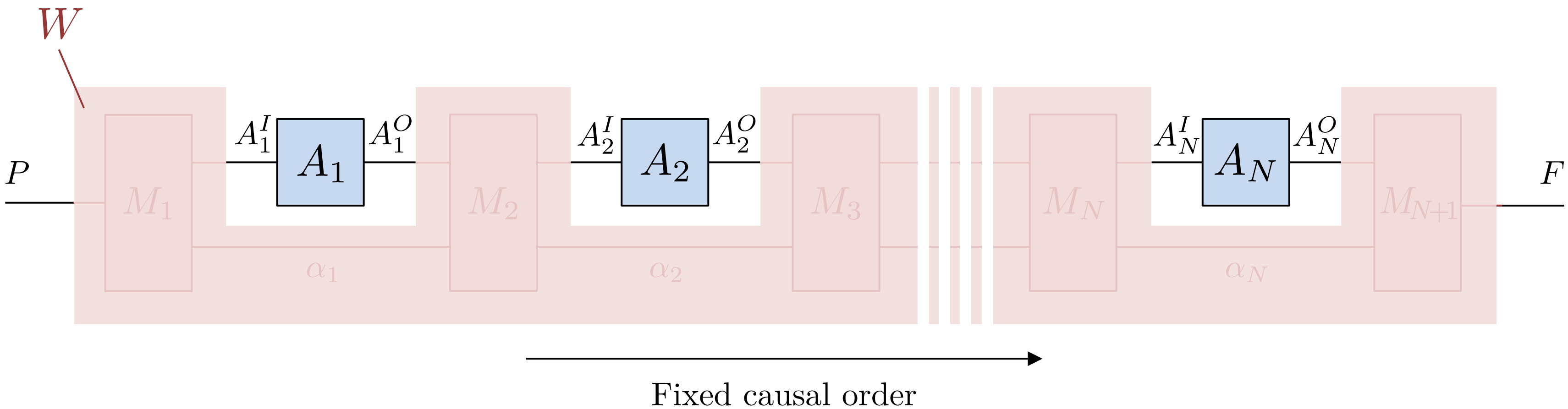
$W \in \mathcal{L}(\mathcal{H}^{P A_1^I A_1^O \cdots A_N^I A_N^O F})$: a "Quantum comb"

[Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

(a "Process matrix" with fixed order)

[Oreshkov, Costa & Brukner, Nat Comms 2012]

Quantum Circuits with Fixed causal Order



$$M = W * \big(A_1 \otimes A_2 \otimes \cdots \otimes A_N\big)$$

with $W = M_1 * M_2 * \cdots * M_N * M_{N+1}$

"Causality" condition:

$$\boxed{W \text{ is a QC-FO} \quad \text{iff} \quad W \geq 0 \text{ and } \forall\, n, \mathrm{Tr}_{A_{n+1}\cdots A_N} W = W_{(n)} \otimes \mathbb{1}^{A_n^O}}$$

(SDP constraints)

[Gutoski & Watrous, STOC 2006; Chiribella, D'Ariano, Perinotti, EPL 2008, PRL 2008, PRA 2009]

Outline

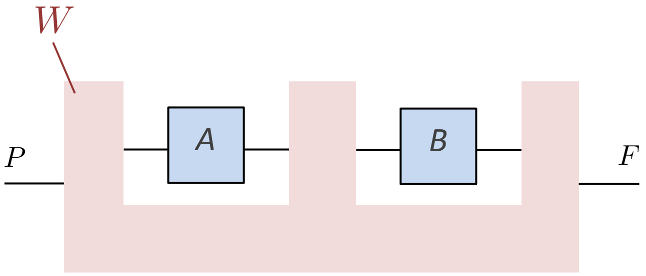
- Quantum Circuits with Fixed causal Order (QC-FOs)

- **Quantum Circuits with Classical Control of causal order** (QC-CCs)

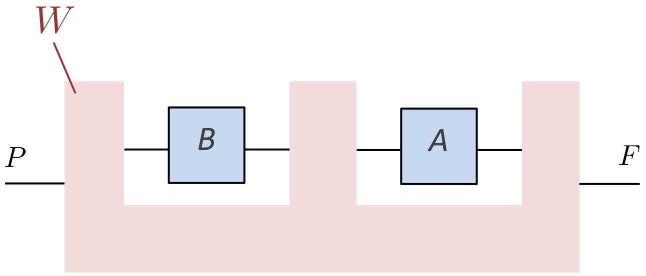
- **Quantum Circuits with Quantum Control of causal order** (QC-QCs)

(Using the process matrix framework)

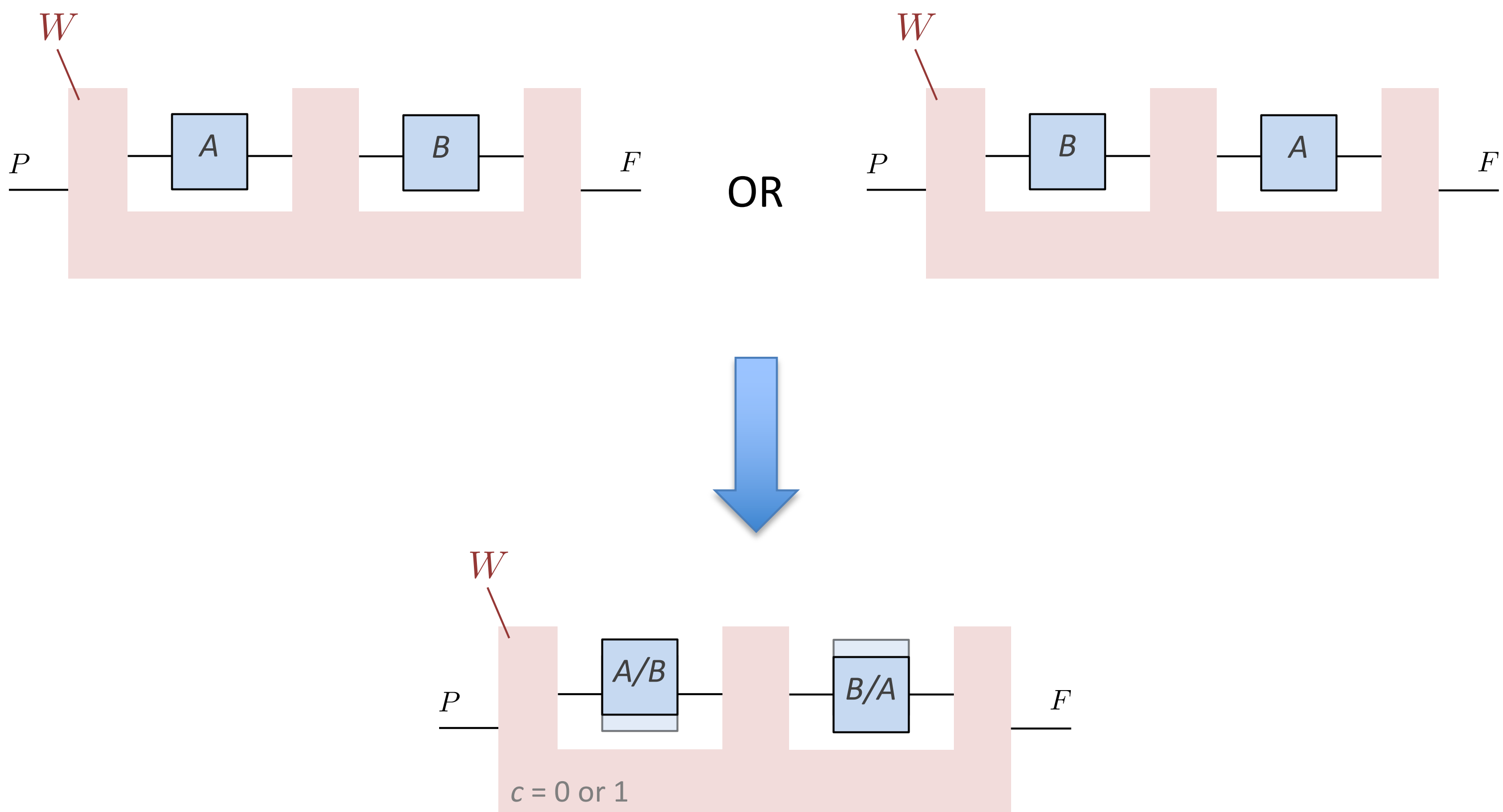
Controlling the causal order?



OR



Controlling the causal order?

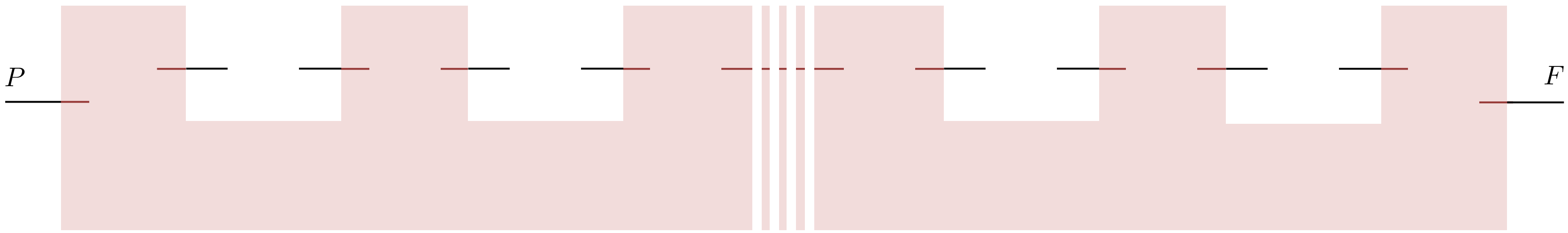


$$\begin{cases} \text{If } c = 0 \text{, apply } A \text{ then } B; \\ \text{If } c = 1 \text{, apply } B \text{ then } A. \end{cases}$$

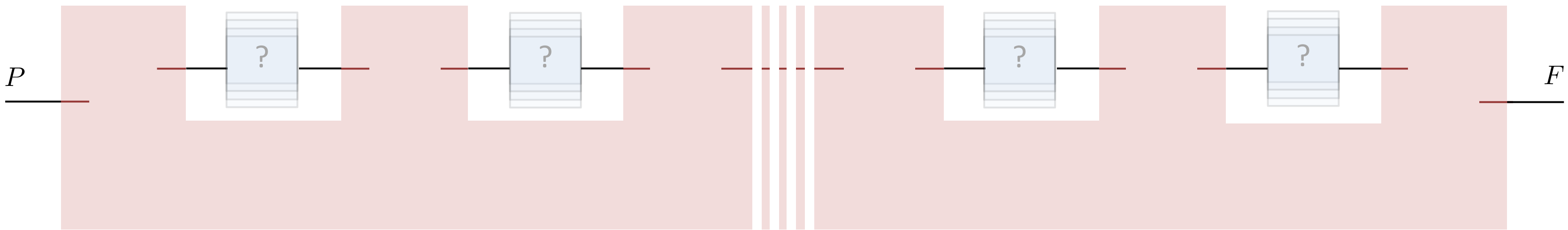
The "classical switch"

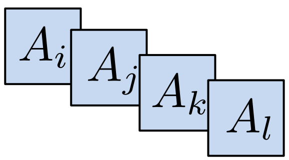
[Chiribella *et al.*, arXiv 2009, PRA 2013]

Quantum **C**ircuits w/ **C**lassical **C**ontrol of causal order

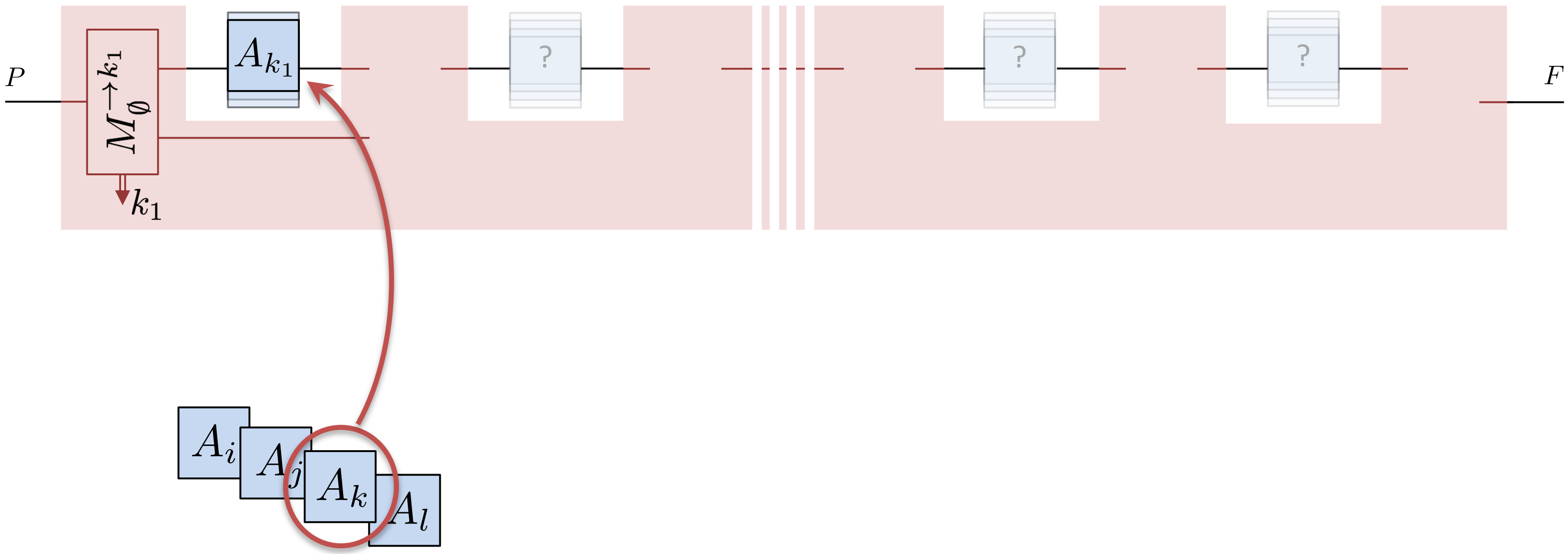


Quantum Circuits w/ Classical Control of causal order

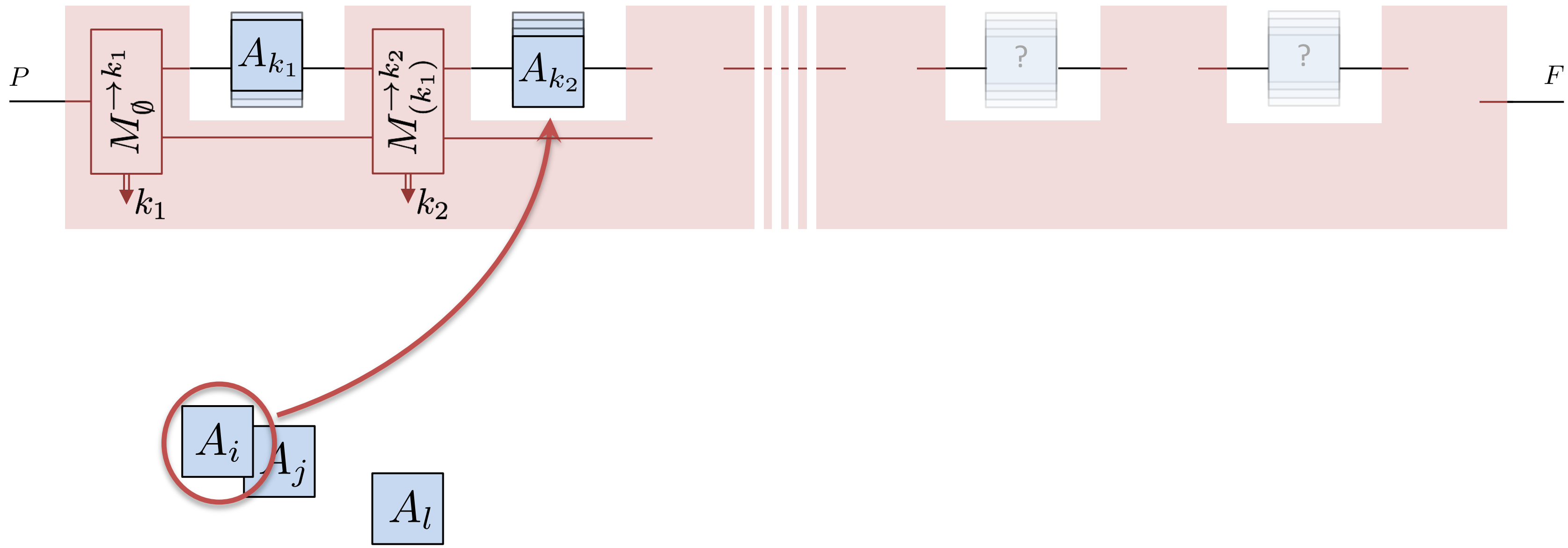




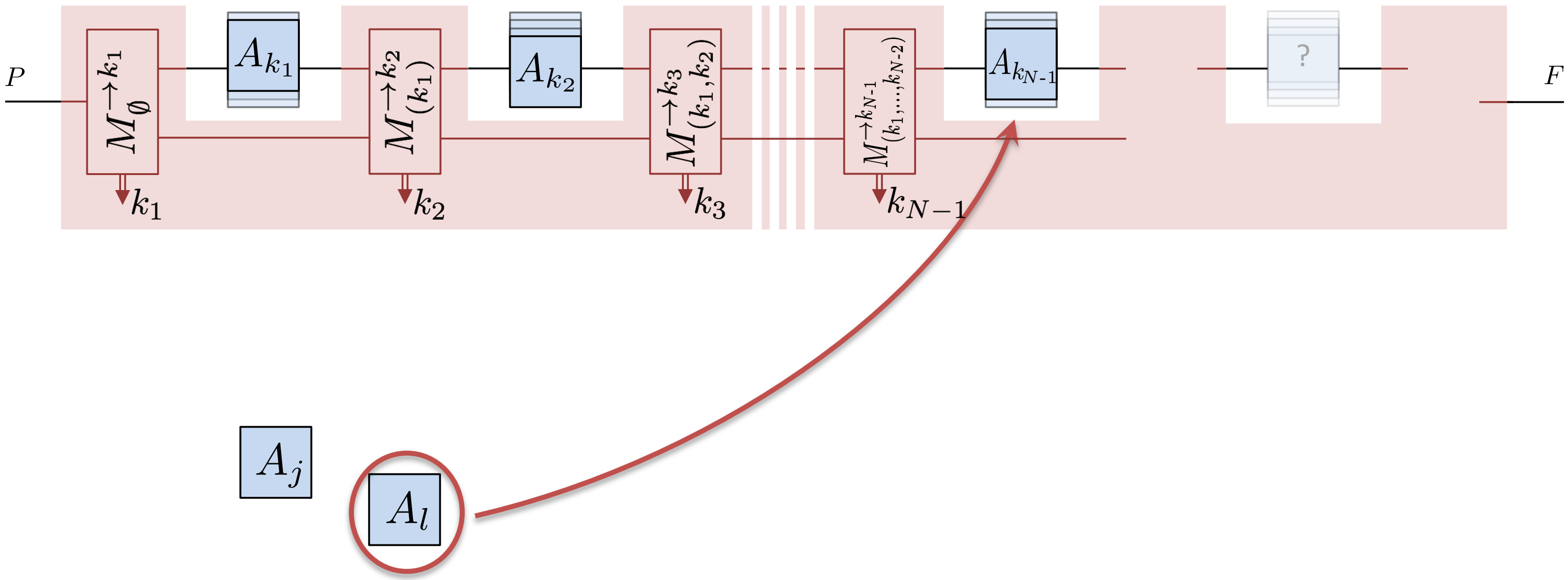
Quantum Circuits w/ Classical Control of causal order



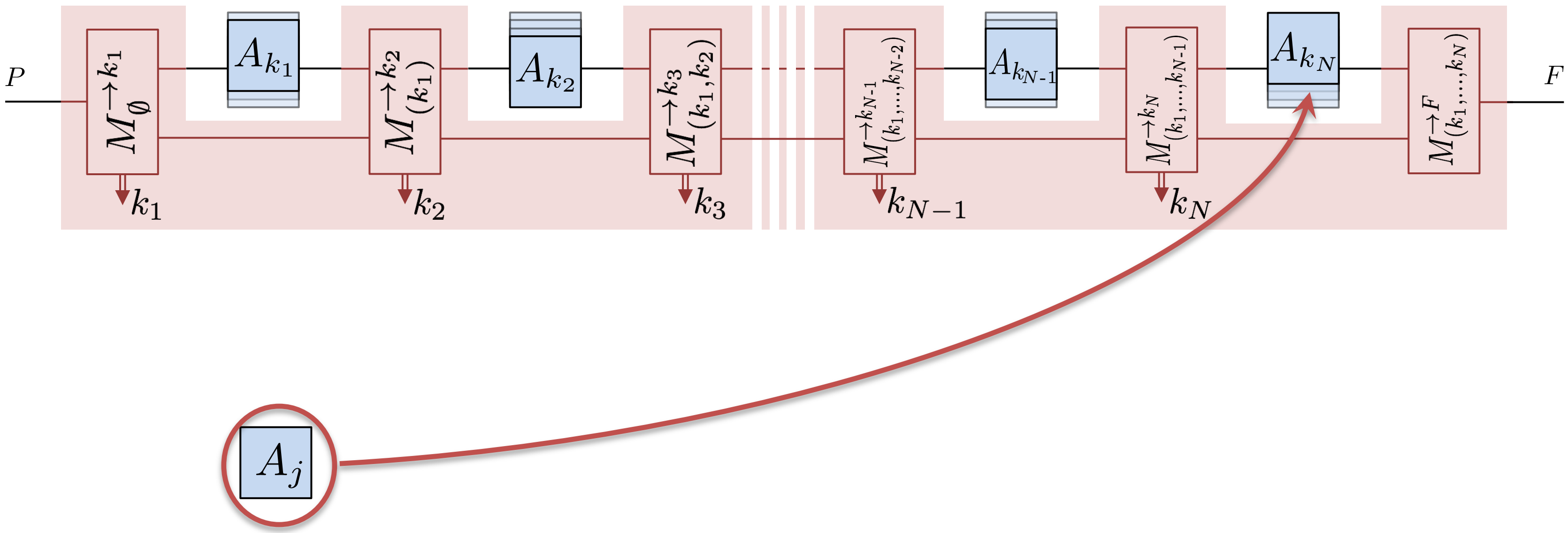
Quantum Circuits w/ Classical Control of causal order



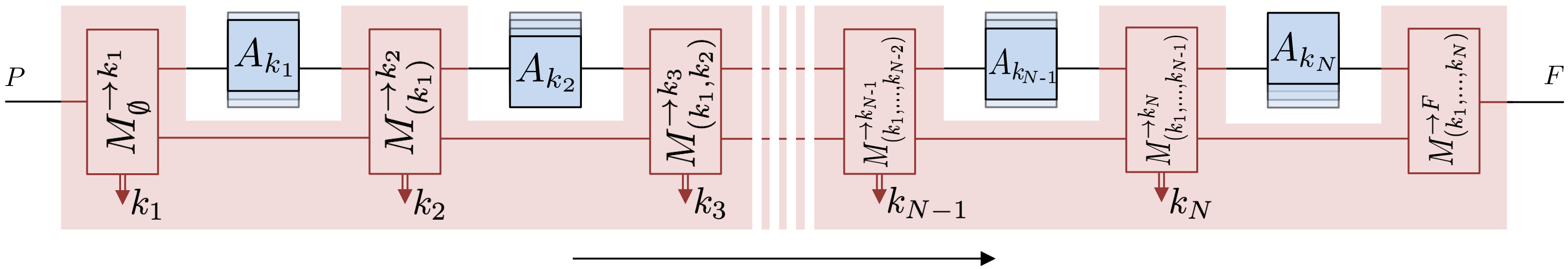
Quantum Circuits w/ Classical Control of causal order



Quantum Circuits w/ Classical Control of causal order

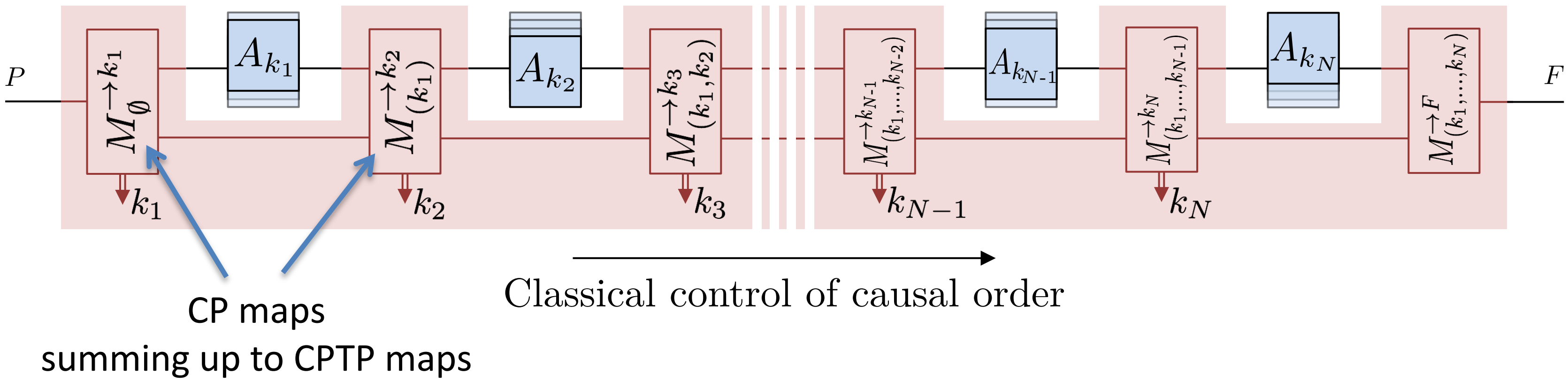


Quantum Circuits w/ Classical Control of causal order

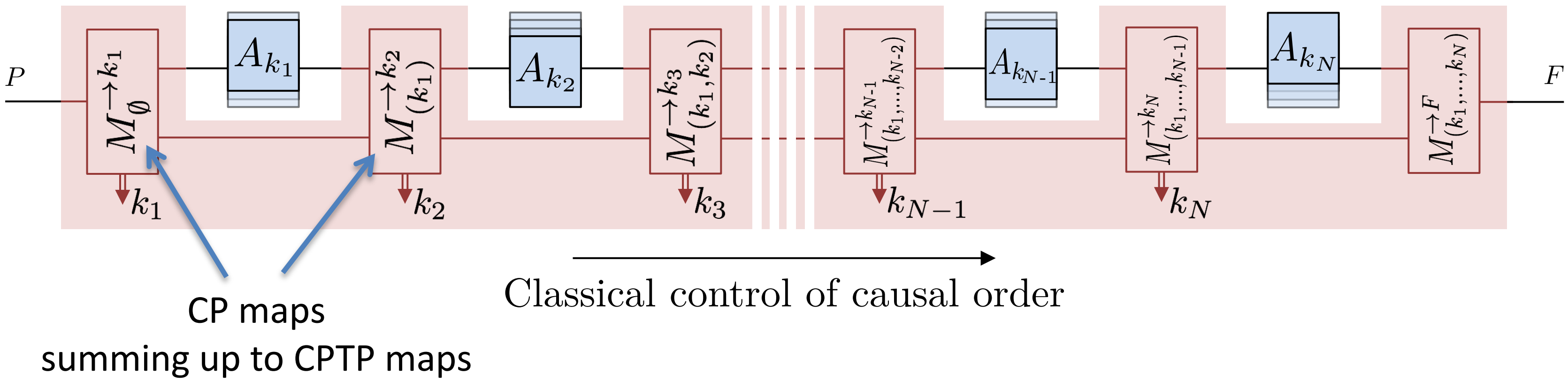


Classical control of causal order

Quantum Circuits w/ Classical Control of causal order



Quantum Circuits w/ Classical Control of causal order

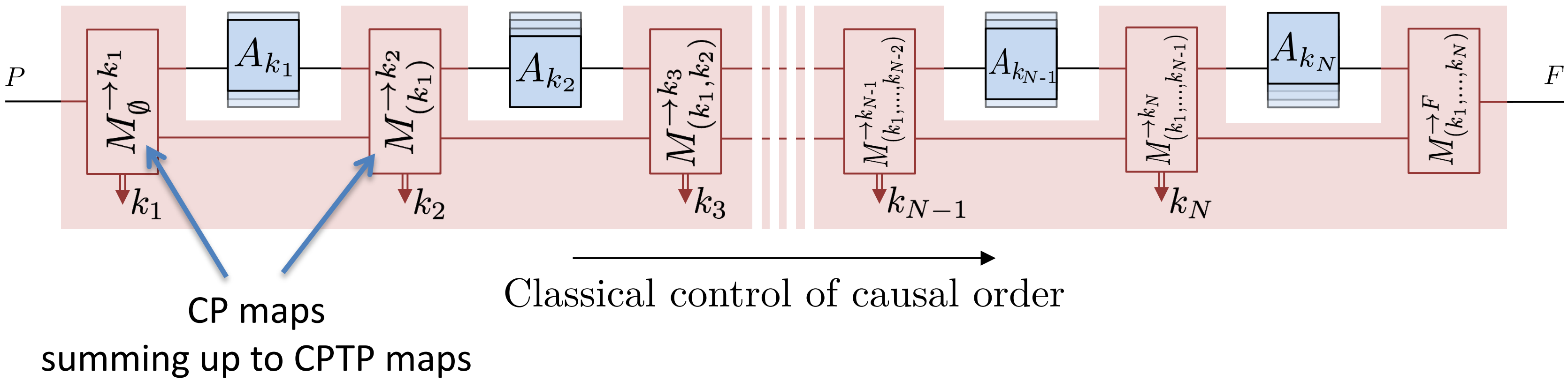


- A classical control decides which operation A_k to be applied next;

  Crucial assumption: each operation is applied **once and only once**

so that it defines a proper "quantum supermap"
& fits in the process matrix framework

Quantum Circuits w/ Classical Control of causal order



- A classical control decides which operation A_k to be applied next;

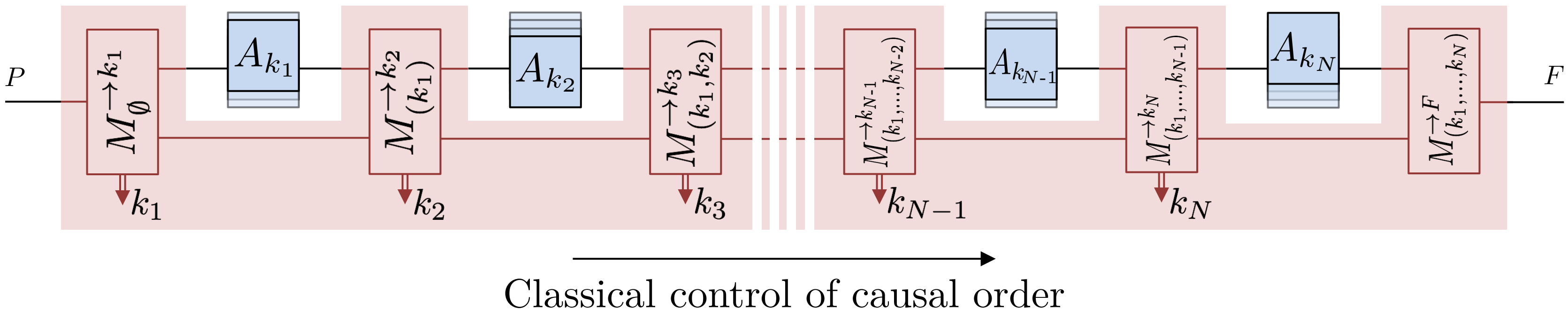
  Crucial assumption: each operation is applied **once and only once**

- The operations order $(k_1,\ldots,k_n)$ is established and recorded on the fly;

  the upcoming operations depend on that order

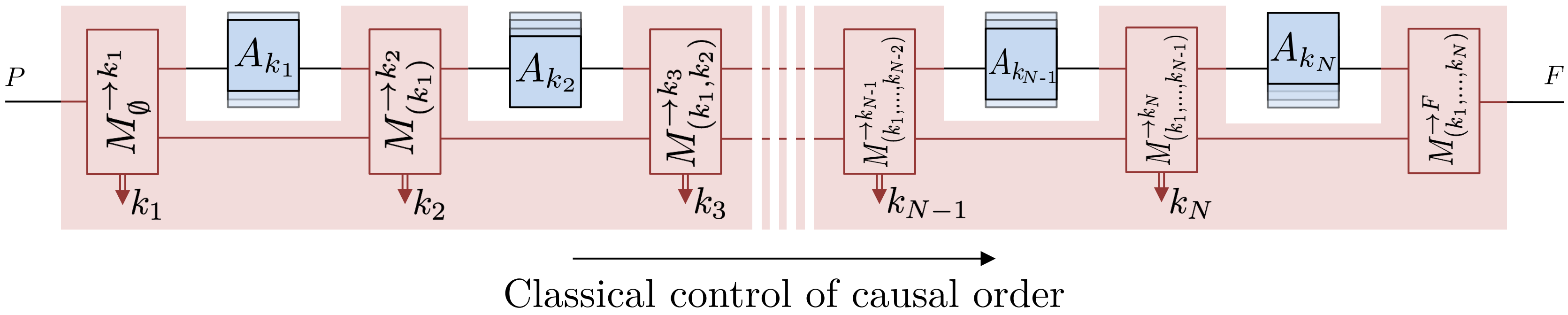
  ➢ "Dynamical causal order"

Quantum **C**ircuits w/ **C**lassical **C**ontrol of causal order



(remember QC-FO case: $W = M_1 * M_2 * \cdots * M_N * M_{N+1}$)

Quantum Circuits w/ Classical Control of causal order

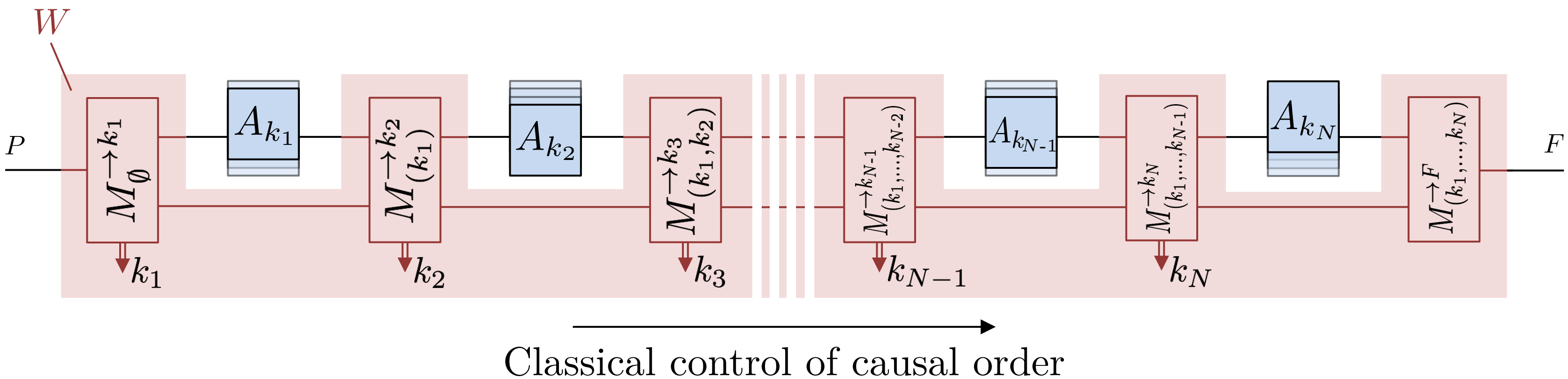


(remember QC-FO case: $W = M_1 * M_2 * \cdots * M_N * M_{N+1}$)

➢ Here:

$$M_{\emptyset}^{\to k_1} * M_{(k_1)}^{\to k_2} * M_{(k_1,k_2)}^{\to k_3} * \cdots * M_{(k_1,\dots,k_{N-1})}^{\to k_N} * M_{(k_1,\dots,k_N)}^{\to F}$$

Quantum Circuits w/ Classical Control of causal order



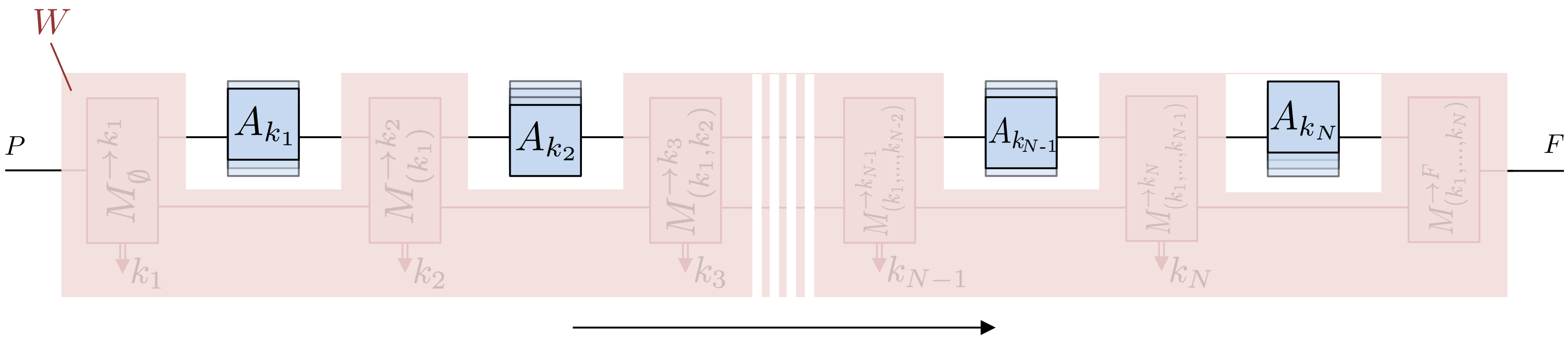
(remember QC-FO case: $W = M_1 * M_2 * \cdots * M_N * M_{N+1}$)

- Here:

$$W = \sum_{(k_1,\dots,k_N)} M_\emptyset^{\to k_1} * M_{(k_1)}^{\to k_2} * M_{(k_1,k_2)}^{\to k_3} * \cdots * M_{(k_1,\dots,k_{N-1})}^{\to k_N} * M_{(k_1,\dots,k_N)}^{\to F}$$

- $M = W * (A_1 \otimes A_2 \otimes \cdots \otimes A_N)$

Quantum Circuits w/ Classical Control of causal order



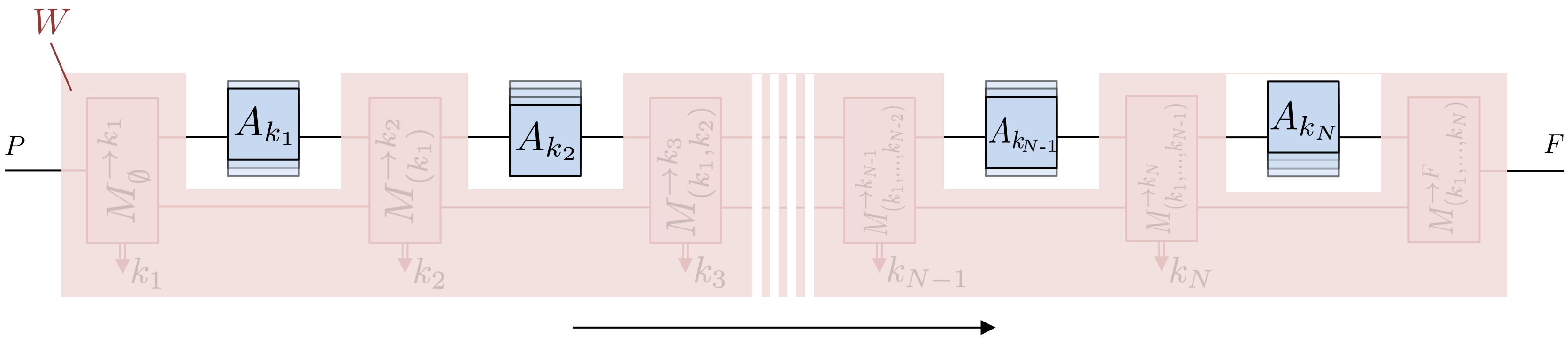
(remember QC-FO case: $W = M_1 * M_2 * \cdots * M_N * M_{N+1}$)

- Here:

$$W = \sum_{(k_1,\dots,k_N)} M_\emptyset^{\to k_1} * M_{(k_1)}^{\to k_2} * M_{(k_1,k_2)}^{\to k_3} * \cdots * M_{(k_1,\dots,k_{N-1})}^{\to k_N} * M_{(k_1,\dots,k_N)}^{\to F}$$

- $M = W * (A_1 \otimes A_2 \otimes \cdots \otimes A_N)$

Quantum Circuits w/ Classical Control of causal order



Classical control of causal order

$$W = \sum_{k_1,\dots,k_N} M_\emptyset^{\to k_1} * M_{(k_1)}^{\to k_2} * \cdots \cdots * M_{(k_1,\dots,k_N)}^{\to F}$$

- $M = W * (A_1 \otimes A_2 \otimes \cdots \otimes A_N)$

W is a QC-CC iff $\exists \{W_{(k_1,\dots,k_n)} \geq 0\}$,

such that $W = \sum_{k_1,\dots,k_N} W_{(k_1,\dots,k_N,F)}$ and

$$\mathrm{Tr}_F W_{(k_1,\dots,k_N,F)} = W_{(k_1,\dots,k_N)} \otimes \mathbb{1}^{A^O_{k_N}},$$

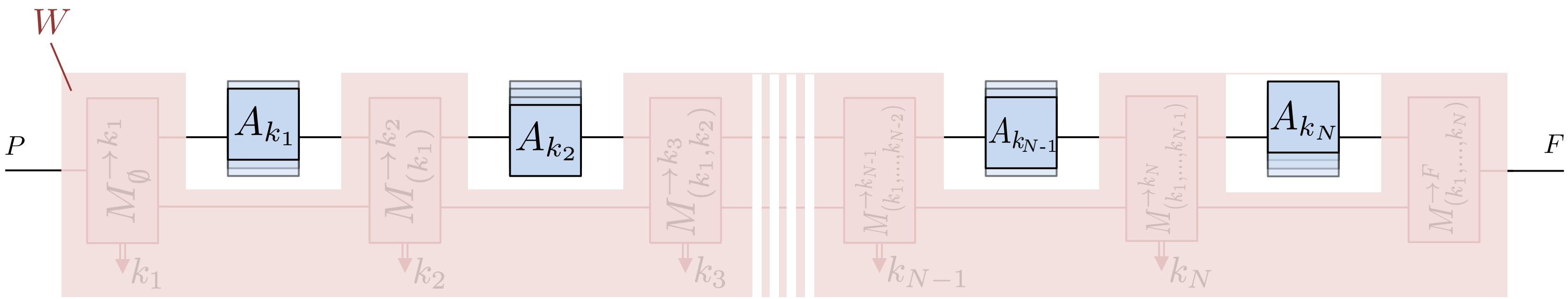
$\forall\, (k_1,\dots,k_n)$,

$$\sum_{k_{n+1}\in\mathcal{N}\backslash\{k_1,\dots,k_n\}} \mathrm{Tr}_{A^I_{k_{n+1}}} W_{(k_1,\dots,k_n,k_{n+1})} = W_{(k_1,\dots,k_n)} \otimes \mathbb{1}^{A^O_{k_n}},$$

and $\sum_{k_1\in\mathcal{N}} \mathrm{Tr}_{A^I_{k_1}} W_{(k_1)} = \mathbb{1}^P$

(SDP constraints)

Quantum Circuits w/ Classical Control of causal order



$$W = \sum_{k_1,\dots,k_N} M_\emptyset^{\to k_1} * M_{(k_1)}^{\to k_2} * \cdots \cdots * M_{(k_1,\dots,k_N)}^{\to F}$$

- $M = W * (A_1 \otimes A_2 \otimes \cdots \otimes A_N)$

W is a QC-CC iff $\exists \{W_{(k_1,\dots,k_n)} \geq 0\}$,

such that $W = \sum_{k_1,\dots,k_N} W_{(k_1,\dots,k_N,F)}$ and

$$\mathrm{Tr}_F W_{(k_1,\dots,k_N,F)} = W_{(k_1,\dots,k_N)} \otimes \mathbb{1}^{A_{k_N}^O},$$

$\forall (k_1,\dots,k_n)$,

$$\sum_{k_{n+1}\in\mathcal{N}\backslash\{k_1,\dots,k_n\}} \mathrm{Tr}_{A_{k_{n+1}}^I} W_{(k_1,\dots,k_n,k_{n+1})} = W_{(k_1,\dots,k_n)} \otimes \mathbb{1}^{A_{k_n}^O},$$

and $\sum_{k_1\in\mathcal{N}} \mathrm{Tr}_{A_{k_1}^I} W_{(k_1)} = \mathbb{1}^P$

(SDP constraints) (causally separable)

[Oreshkov, Costa, Brukner, Nat. Commun. 2012; Oreshkov & Giarmatzi, NJP 2016; Wechs, Abbott, CB, NJP2019]

Outline

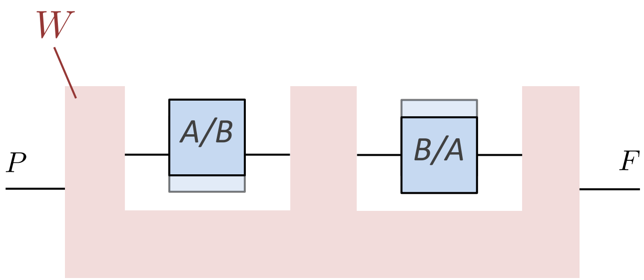
- Quantum Circuits with Fixed causal Order (QC-FOs)

- **Quantum Circuits with Classical Control of causal order** (QC-CCs)

- **Quantum Circuits with Quantum Control of causal order** (QC-QCs)

(Using the process matrix framework)

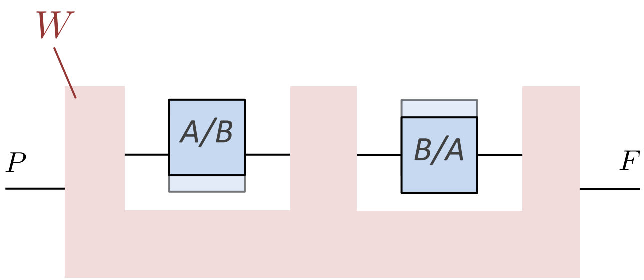
The "Classical switch"



$$\begin{cases} \text{If } c = 0, \text{ apply } A \text{ then } B; \\ \text{If } c = 1, \text{ apply } B \text{ then } A. \end{cases}$$

[Chiribella *et al.*, arXiv 2009, PRA 2013]

The ~~"Classical~~ Quantum switch"



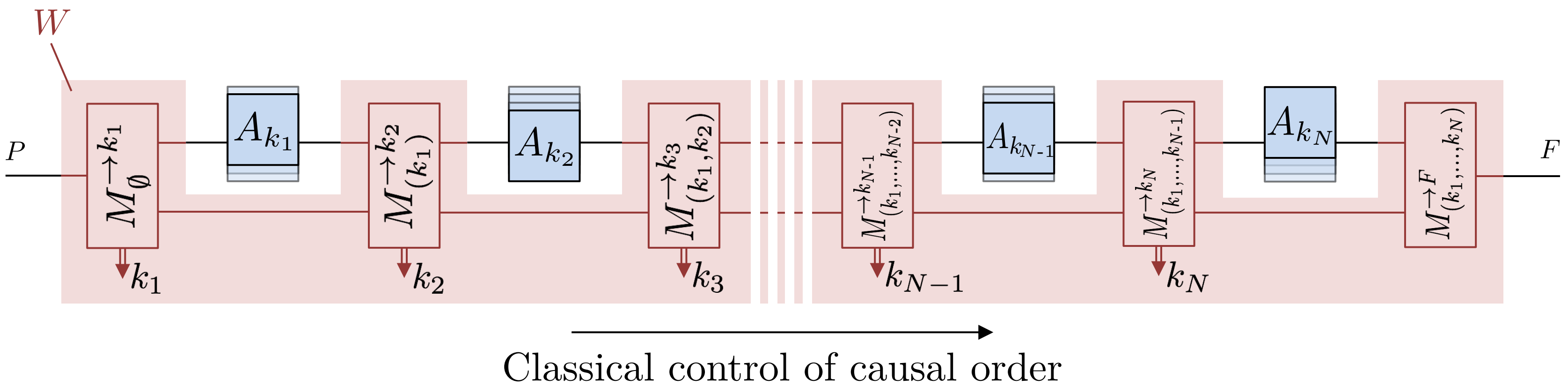
~~If c = 0, apply A then B;~~
~~If c = 1, apply B then A.~~

$$\begin{cases} |0\rangle^c \otimes |\psi\rangle^t \to |0\rangle^c \otimes BA|\psi\rangle^t \\ |1\rangle^c \otimes |\psi\rangle^t \to |1\rangle^c \otimes AB|\psi\rangle^t \end{cases}$$

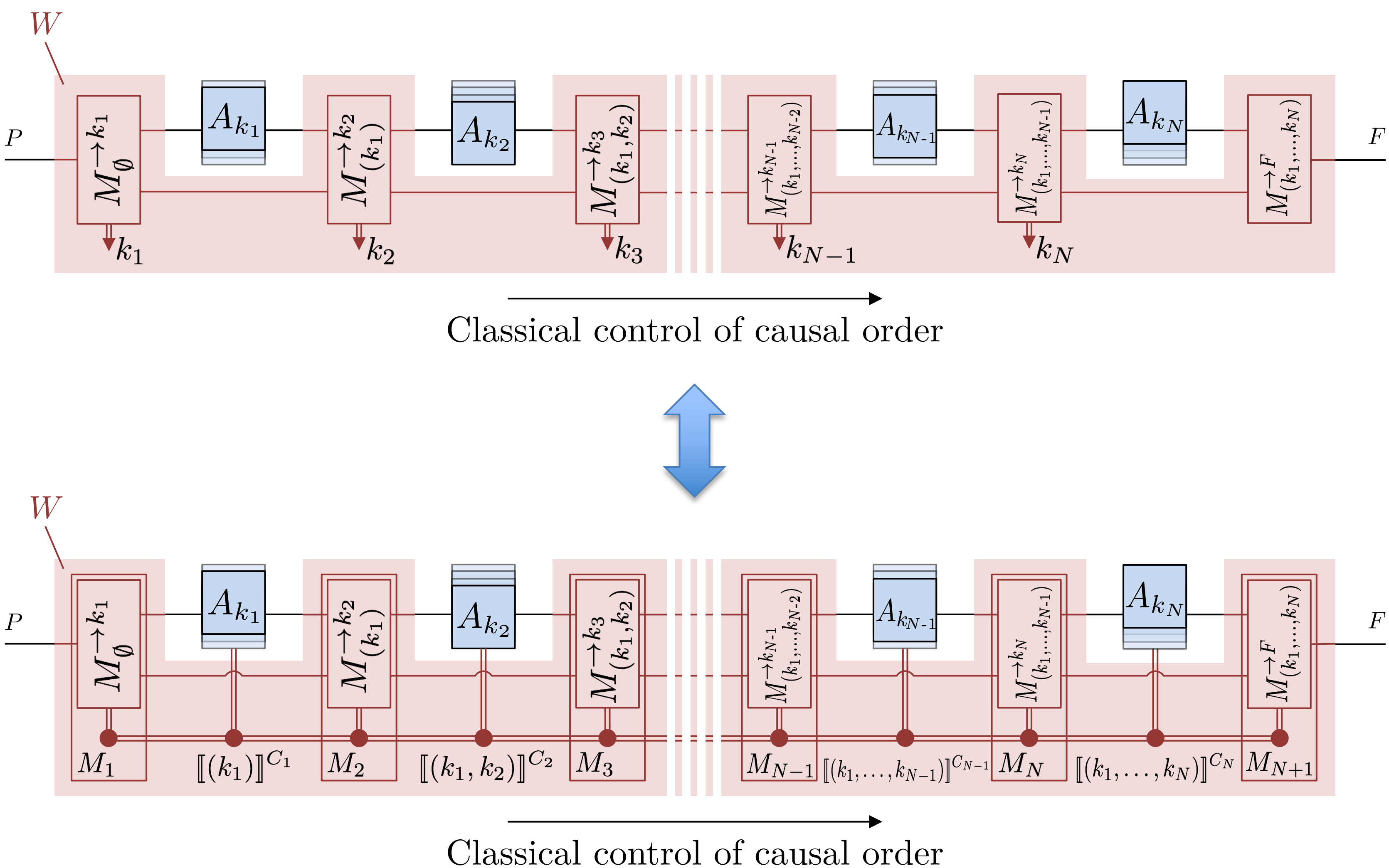
$$(|0\rangle^c + |1\rangle^c) \otimes |\psi\rangle^t \to |0\rangle^c \otimes BA|\psi\rangle^t + |1\rangle^c \otimes AB|\psi\rangle^t$$

[Chiribella *et al.*, arXiv 2009, PRA 2013]

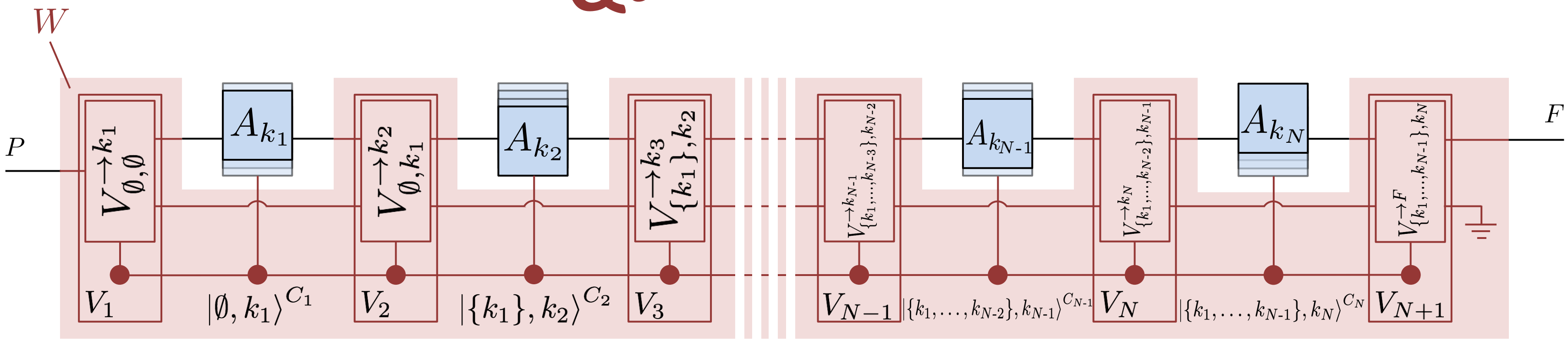
Quantum Circuits w/ Classical Control of causal order



Quantum Circuits w/ Classical Control of causal order

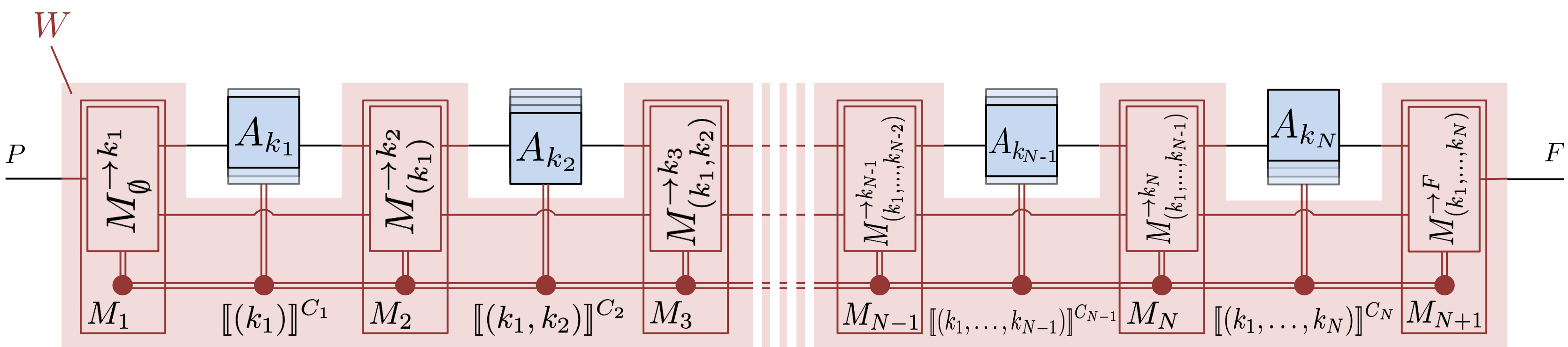


Quantum Circuits w/ ~~Classical~~ Quantum Control of causal order



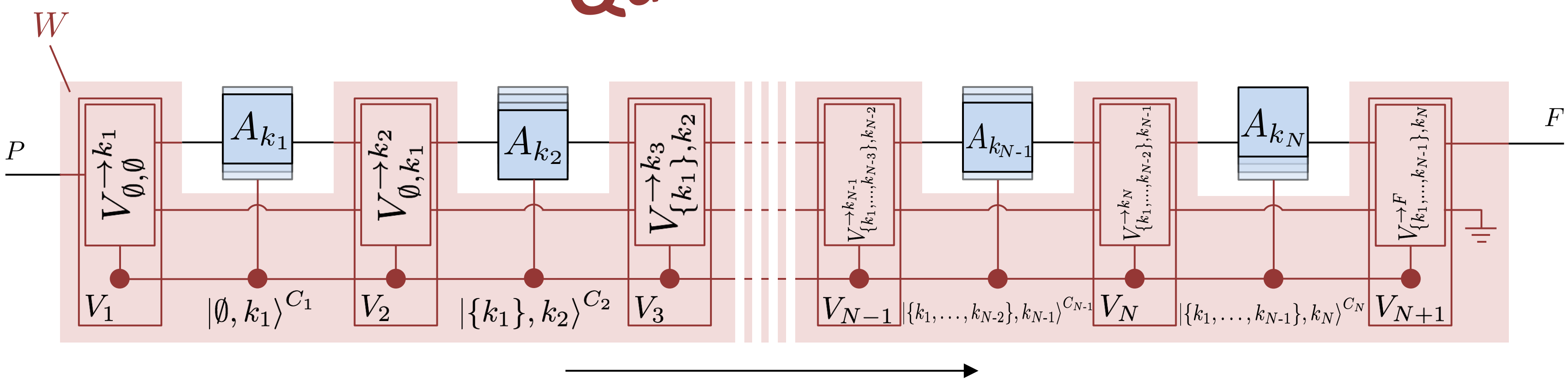
Quantum control of causal order

Make the control quantum!



Classical control of causal order

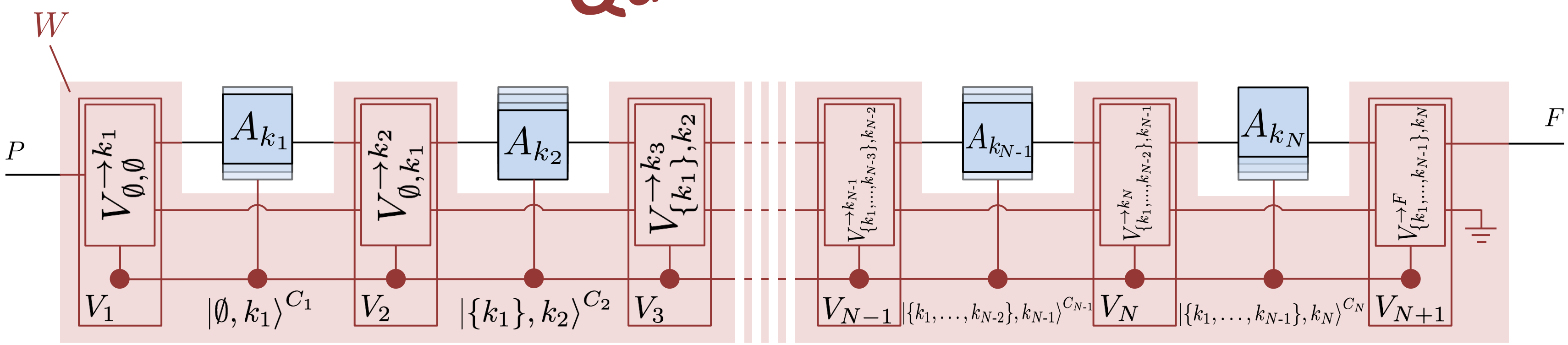
Quantum Circuits w/ ~~Classical~~ Quantum Control of causal order



Quantum control of causal order

- To turn the classical control into a *quantum control*:
  - Operations are made coherent
  - Rather than the whole causal order ($k_1,\ldots,k_n$), the control system only records the currently applied operation k_n, and the unordered set $\{k_1,\ldots,k_{n-1}\}$ of operations applied before

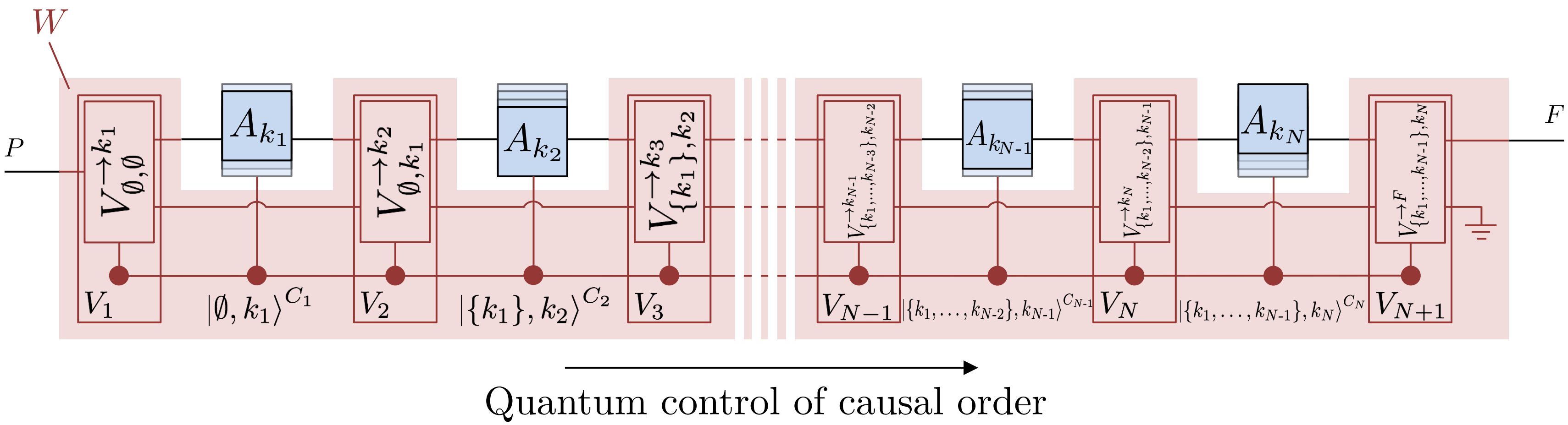
Quantum Circuits w/ ~~Classical~~ Quantum Control of causal order



- To turn the classical control into a *quantum control*:
  - Operations are made coherent
  - Rather than the whole causal order ($k_1,\ldots,k_n$), the control system only records the currently applied operation k_n, and the unordered set $\{k_1,\ldots,k_{n-1}\}$ of operations applied before
    - Allows for different orders to interfere
      - "causally nonseparable processes"

[Oreshkov, Costa, Brukner, Nat. Commun. 2012; Oreshkov & Giarmatzi, NJP 2016; Wechs, Abbott, CB, NJP2019]

Quantum Circuits w/ Quantum Control of causal order

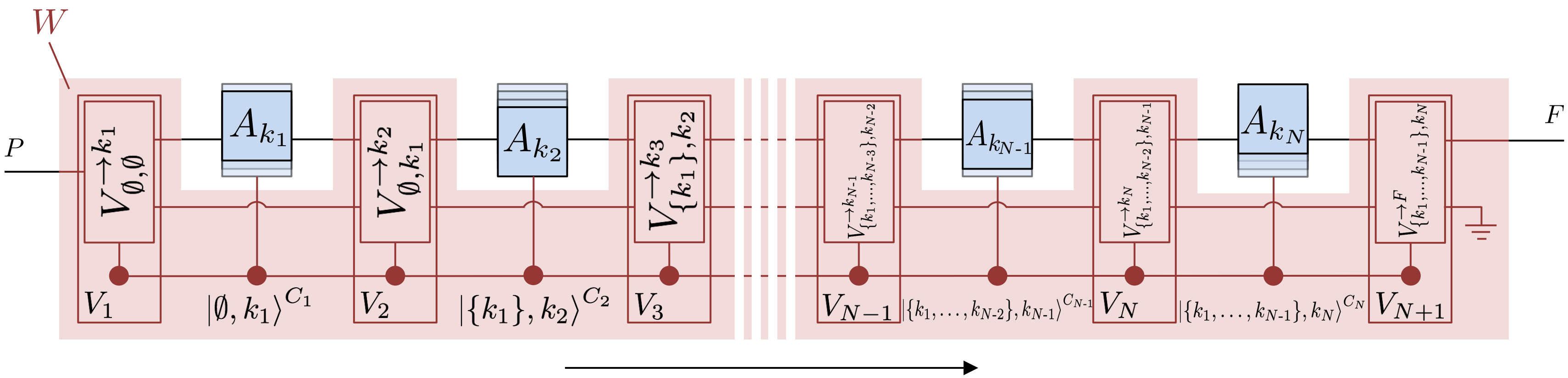


(remember QC-CC case:

$$W = \sum_{(k_1,\ldots,k_N)} M_{\emptyset}^{\to k_1} * M_{(k_1)}^{\to k_2} * M_{(k_1,k_2)}^{\to k_3} * \cdots * M_{(k_1,\ldots,k_{N-1})}^{\to k_N} * M_{(k_1,\ldots,k_N)}^{\to F}$$

)

Quantum Circuits w/ Quantum Control of causal order



(remember QC-CC case:

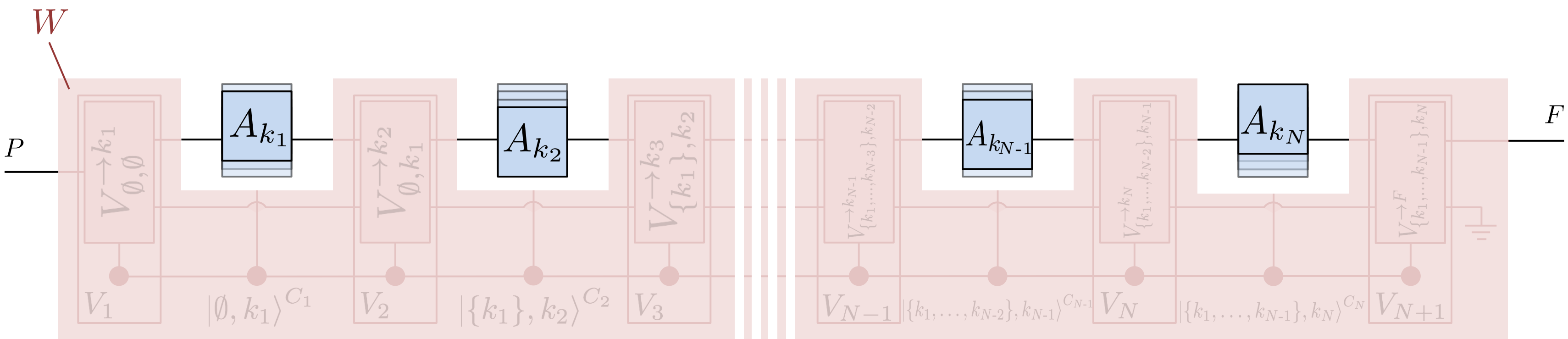
$$W = \sum_{(k_1,\ldots,k_N)} M_{\emptyset}^{\to k_1} * M_{(k_1)}^{\to k_2} * M_{(k_1,k_2)}^{\to k_3} * \cdots * M_{(k_1,\ldots,k_{N-1})}^{\to k_N} * M_{(k_1,\ldots,k_N)}^{\to F} \;)$$

➢ Here:

$$|w\rangle = \sum_{(k_1,\ldots,k_N)} |V_{\emptyset,\emptyset}^{\to k_1}\rangle\!\rangle * |V_{\emptyset,k_1}^{\to k_2}\rangle\!\rangle * |V_{\{k_1\},k_2}^{\to k_3}\rangle\!\rangle * \cdots * |V_{\{k_1,\ldots,k_{N-2}\},k_{N-1}}^{\to k_N}\rangle\!\rangle * |V_{\{k_1,\ldots,k_{N-1}\},k_N}^{\to F}\rangle\!\rangle$$

➢ $W = \mathrm{Tr}_{\text{anc.}} |w\rangle\langle w| \,, \quad M = W * (A_1 \otimes A_2 \otimes \cdots \otimes A_N)$

Quantum Circuits w/ Quantum Control of causal order



(remember QC-CC case:

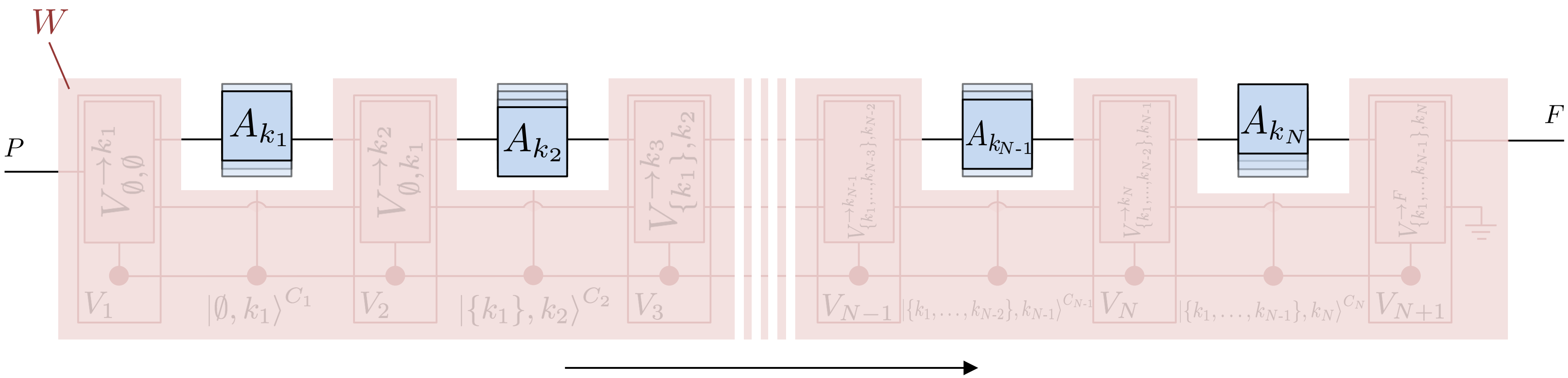
$$W = \sum_{(k_1,\ldots,k_N)} M_{\emptyset}^{\to k_1} * M_{(k_1)}^{\to k_2} * M_{(k_1,k_2)}^{\to k_3} * \cdots * M_{(k_1,\ldots,k_{N-1})}^{\to k_N} * M_{(k_1,\ldots,k_N)}^{\to F} \;)$$

- Here:

$$|w\rangle = \sum_{(k_1,\ldots,k_N)} |V_{\emptyset,\emptyset}^{\to k_1}\rangle\!\rangle * |V_{\emptyset,k_1}^{\to k_2}\rangle\!\rangle * |V_{\{k_1\},k_2}^{\to k_3}\rangle\!\rangle * \cdots * |V_{\{k_1,\ldots,k_{N-2}\},k_{N-1}}^{\to k_N}\rangle\!\rangle * |V_{\{k_1,\ldots,k_{N-1}\},k_N}^{\to F}\rangle\!\rangle$$

- $$W = \mathrm{Tr}_{\mathrm{anc.}} |w\rangle\langle w| \;, \qquad M = W * (A_1 \otimes A_2 \otimes \cdots \otimes A_N)$$

Quantum Circuits w/ Quantum Control of causal order



$$W = \mathrm{Tr}_{\mathrm{anc.}} |w\rangle\rangle\langle\langle w|$$

with $|w\rangle\rangle = \sum_{k_1,\dots,k_N} |V_{\emptyset,\emptyset}^{\to k_1}\rangle\rangle * |V_{\emptyset,k_1}^{\to k_2}\rangle\rangle * \cdots$
$\cdots * |V_{\{k_1,\dots,k_{N-1}\},k_N}^{\to F}\rangle\rangle$

➢ $M = W * (A_1 \otimes A_2 \otimes \cdots \otimes A_N)$

W is a QC-QC iff $\exists \{W_{(\mathcal{K},\ell)} \geq 0\}$,

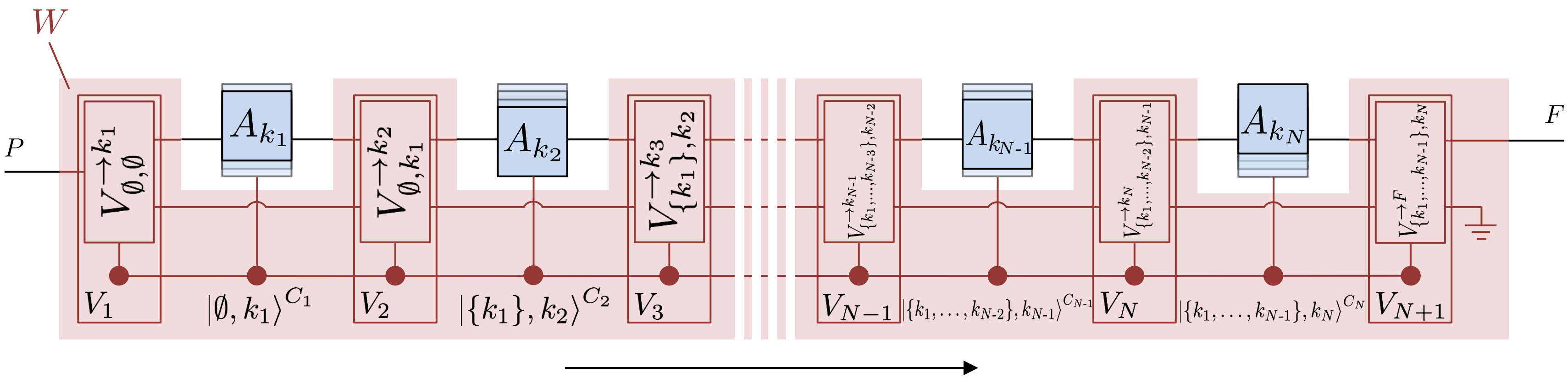
$$\mathrm{Tr}_F W = \sum_{k\in\mathcal{N}} W_{(\mathcal{N}\setminus\{k\},k)} \otimes \mathbb{1}^{A_k^O},$$

$$\forall \emptyset \subsetneq \mathcal{K} \subsetneq \mathcal{N}, \sum_{\ell\in\mathcal{N}\setminus\mathcal{K}} \mathrm{Tr}_{A_\ell^I} W_{(\mathcal{K},\ell)} = \sum_{k\in\mathcal{K}} W_{(\mathcal{K}\setminus\{k\},k)} \otimes \mathbb{1}^{A_k^O},$$

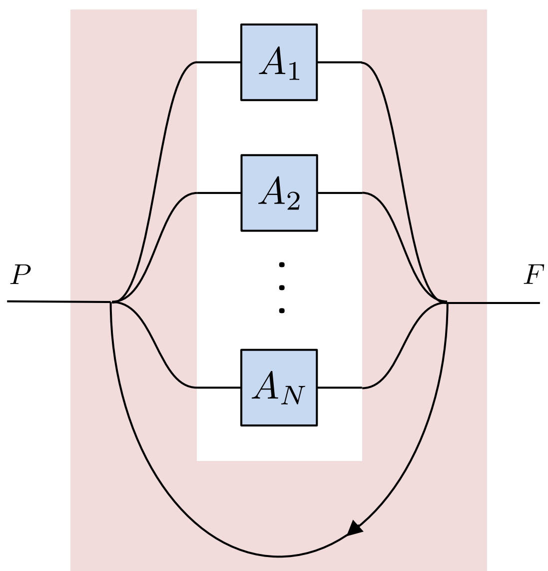
$$\text{and} \quad \sum_{\ell\in\mathcal{N}} \mathrm{Tr}_{A_\ell^I} W_{(\emptyset,\ell)} = \mathbb{1}^P.$$

(SDP constraints)

"Realisations" of QC-QCs?

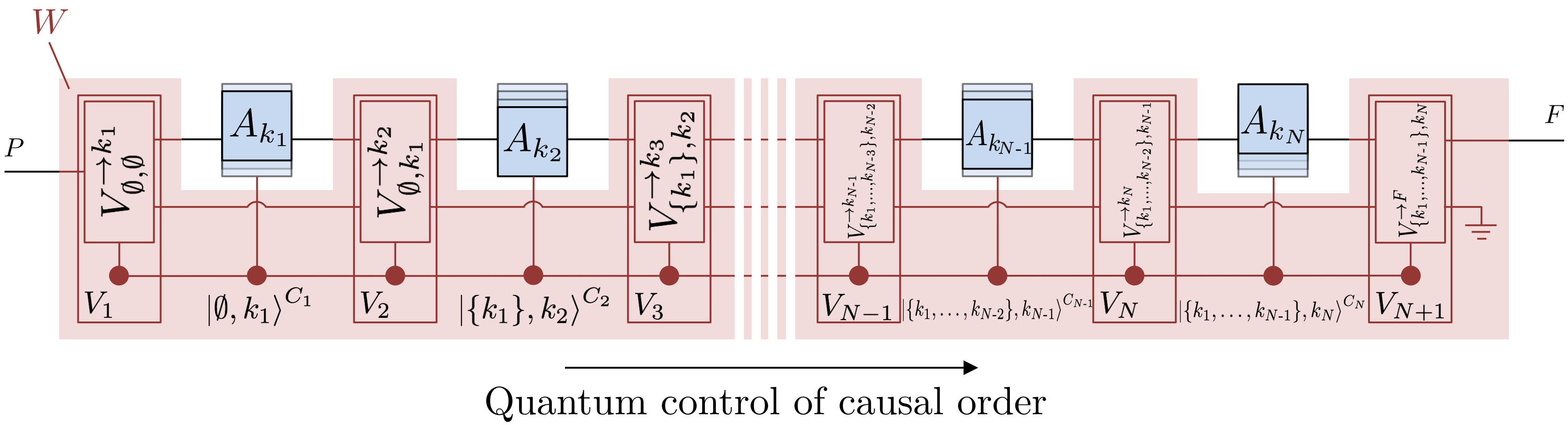


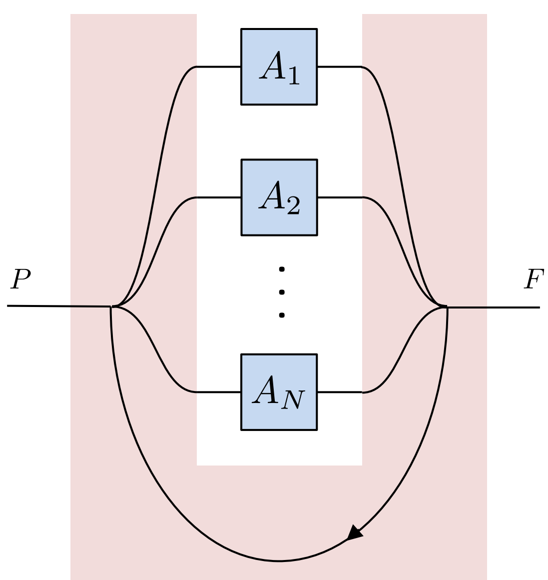
Quantum control of causal order



- "Target system"
  = some d.o.f. of a travelling particle
- Control states:
  $$|\mathcal{K}_{n-1}, k_n\rangle^{C_n} = |\mathcal{K}_{n-1}\rangle^{\substack{\text{some other}\\ \text{d.o.f.}}} \otimes |k_n\rangle^{\text{path}}$$
- Circuit should "route" the particle
  and apply the internal operations *V*

"Realisations" of QC-QCs?





- "Target system" = some d.o.f. of a travelling particle
- Control states:

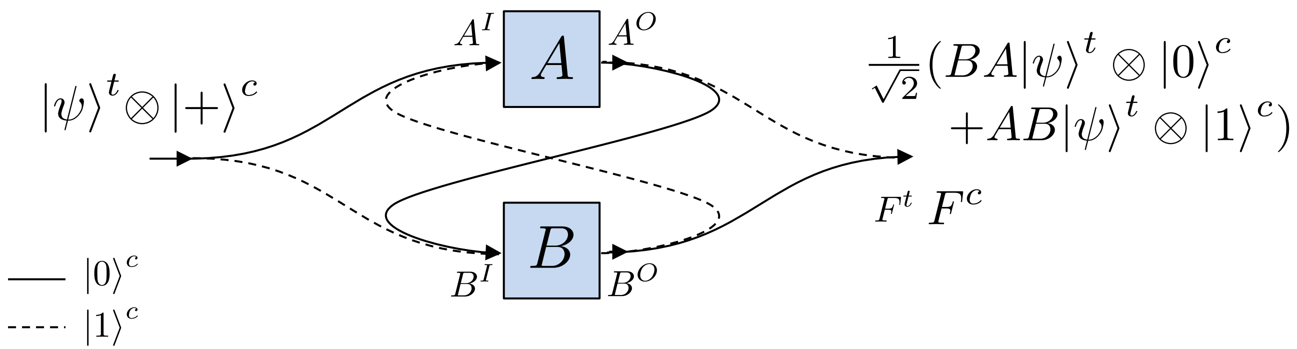
$$|\mathcal{K}_{n-1}, k_n\rangle^{C_n} = |\mathcal{K}_{n-1}\rangle^{\substack{\text{some other}\\ \text{d.o.f.}}} \otimes |k_n\rangle^{\text{path}}$$

- Circuit should "route" the particle and apply the internal operations *V*

Time-dependent
or using a "timer" system
to be incremented

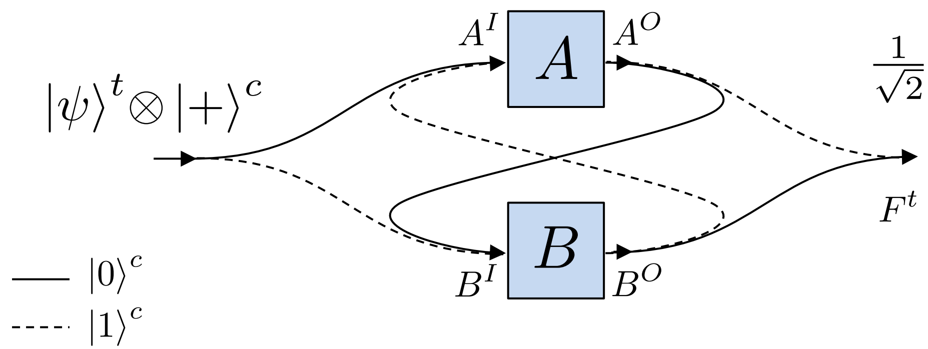
The Quantum switch

[Chiribella *et al.*, arXiv 2009, PRA 2013]

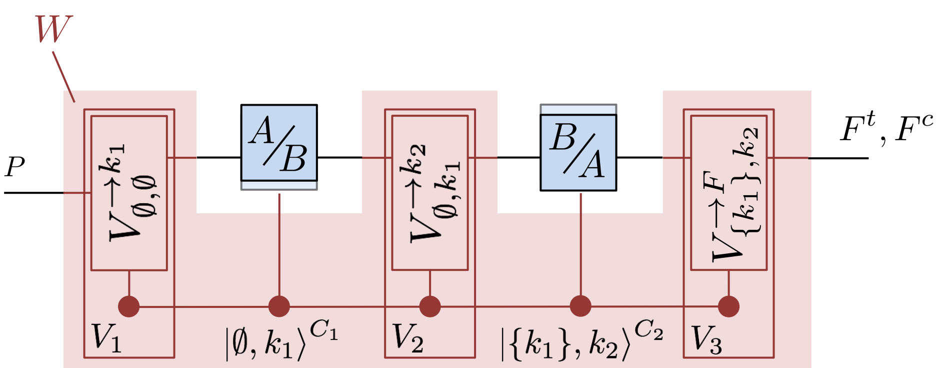


The Quantum switch

[Chiribella *et al.*, arXiv 2009, PRA 2013]



$$\frac{1}{\sqrt{2}}\left(BA|\psi\rangle^t \otimes |0\rangle^c + AB|\psi\rangle^t \otimes |1\rangle^c\right)$$



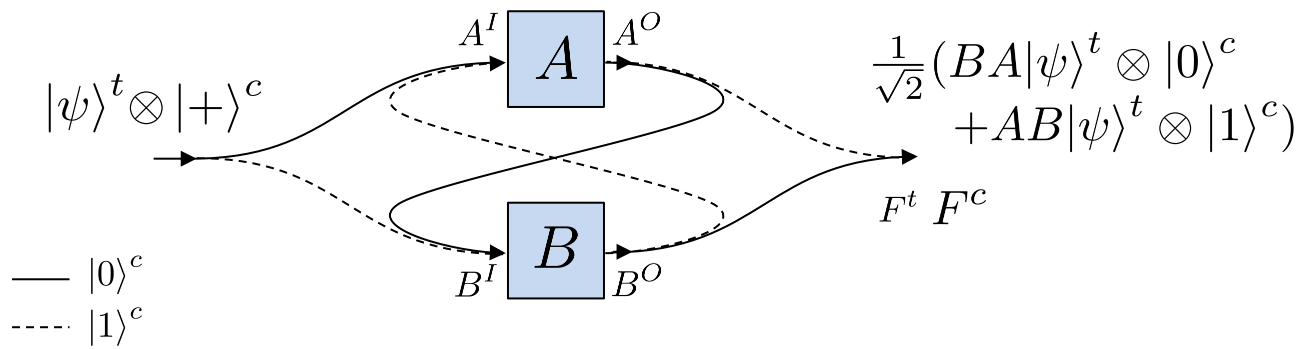
$$|V^{\to A}_{\emptyset,\emptyset}\rangle\rangle = \frac{1}{\sqrt{2}}|\psi\rangle^{A^I}, \quad |V^{\to B}_{\emptyset,\emptyset}\rangle\rangle = \frac{1}{\sqrt{2}}|\psi\rangle^{B^I}$$

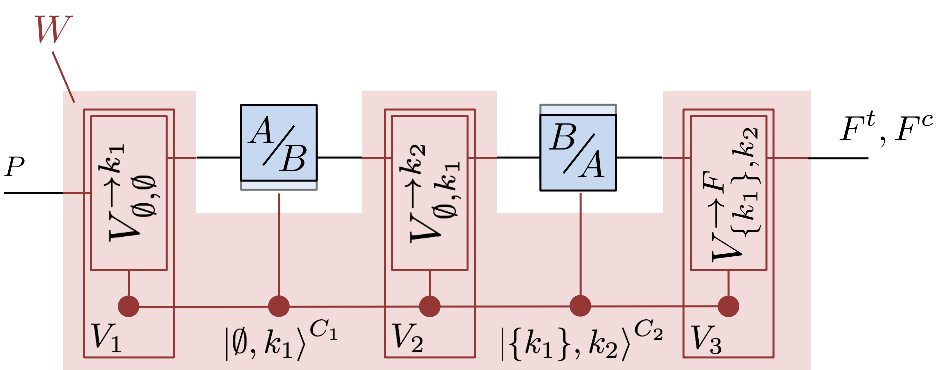
$$|V^{\to B}_{\emptyset,A}\rangle\rangle = |\mathbb{I}\rangle\rangle^{A^O B^I}, \quad |V^{\to A}_{\emptyset,B}\rangle\rangle = |\mathbb{I}\rangle\rangle^{B^O A^I}$$

$$|V^{\to F}_{\{A\},B}\rangle\rangle = |\mathbb{I}\rangle\rangle^{B^O F^t}|0\rangle^{F^c}, \quad |V^{\to F}_{\{B\},A}\rangle\rangle = |\mathbb{I}\rangle\rangle^{A^O F^t}|1\rangle^{F^c}$$

The Quantum switch

[Chiribella *et al.*, arXiv 2009, PRA 2013]





$$|V^{\to A}_{\emptyset,\emptyset}\rangle\rangle = \frac{1}{\sqrt{2}}|\psi\rangle^{A^I}, \quad |V^{\to B}_{\emptyset,\emptyset}\rangle\rangle = \frac{1}{\sqrt{2}}|\psi\rangle^{B^I}$$

$$|V^{\to B}_{\emptyset,A}\rangle\rangle = |\mathbb{I}\rangle\rangle^{A^O B^I}, \quad |V^{\to A}_{\emptyset,B}\rangle\rangle = |\mathbb{I}\rangle\rangle^{B^O A^I}$$

$$|V^{\to F}_{\{A\},B}\rangle\rangle = |\mathbb{I}\rangle\rangle^{B^O F^t}|0\rangle^{F^c}, \quad |V^{\to F}_{\{B\},A}\rangle\rangle = |\mathbb{I}\rangle\rangle^{A^O F^t}|1\rangle^{F^c}$$

$$\Rightarrow \quad W = |w\rangle\langle w| \ \text{ with } \ |w\rangle = \frac{1}{\sqrt{2}}\Big(|\psi\rangle^{A^I}|\mathbb{1}\rangle\rangle^{A^O B^I}|\mathbb{1}\rangle\rangle^{B^O F^t}|0\rangle^{F^c} + |\psi\rangle^{B^I}|\mathbb{1}\rangle\rangle^{B^O A^I}|\mathbb{1}\rangle\rangle^{A^O F^t}|1\rangle^{F^c}\Big)$$

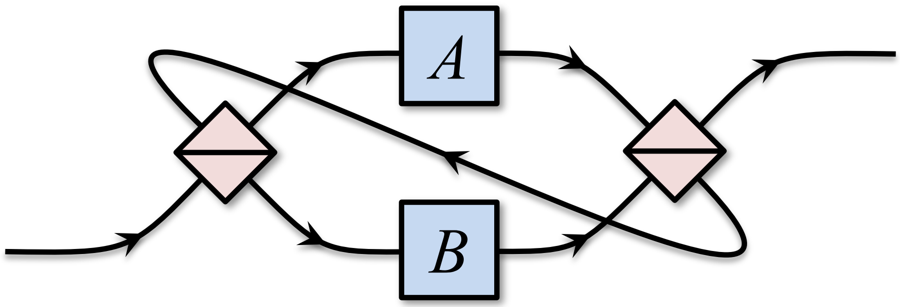
[Araújo *et al.*, NJP 2015; Oreshkov & Giarmatzi, NJP 2016]

The Quantum switch

[Chiribella *et al.*, arXiv 2009, PRA 2013]

- One possible implementation:

[Araújo *et al.*, PRL 2014]



- "Target system"
  = some internal d.o.f.
  of the photon (e.g. OAM)

- Control system $|\cdot\rangle^{C_n}$:
  polarisation + path

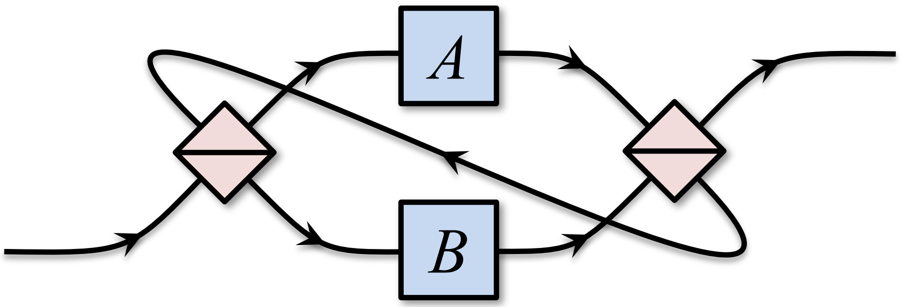
[Experiments:
Procopio *et al.*, Nat. Commun. 2015;
Rubino *et al.*, Sci. Adv. 2017; Rubino *et al.*, arXiv 2017;
Goswami *et al.*, PRL 2018; Goswami *et al.*, arXiv 2018;
Wei *et al.*, PRL 2019; Guo *et al.*, arXiv 2018]

The Quantum switch

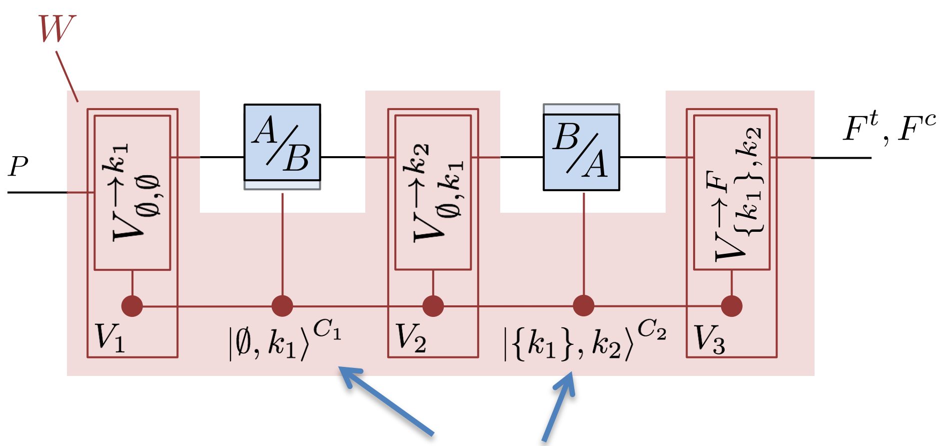
[Chiribella *et al.*, arXiv 2009, PRA 2013]

- One possible implementation:

[Araújo *et al.*, PRL 2014]



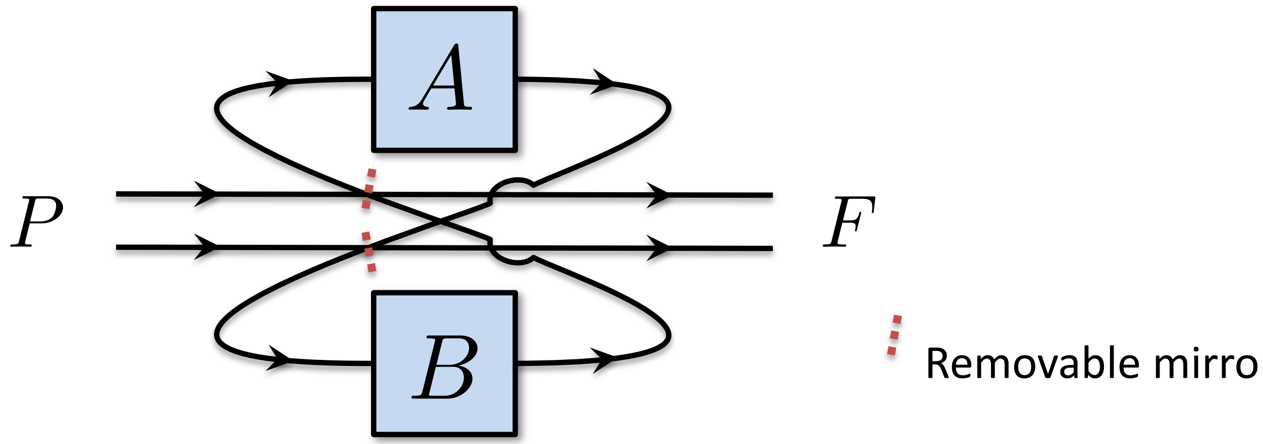
- "Target system" = some internal d.o.f. of the photon (e.g. OAM)
- Control system $|\cdot\rangle^{C_n}$: polarisation + path



Actually only need to be 2-dimensional
(if time-dependent *V*s)

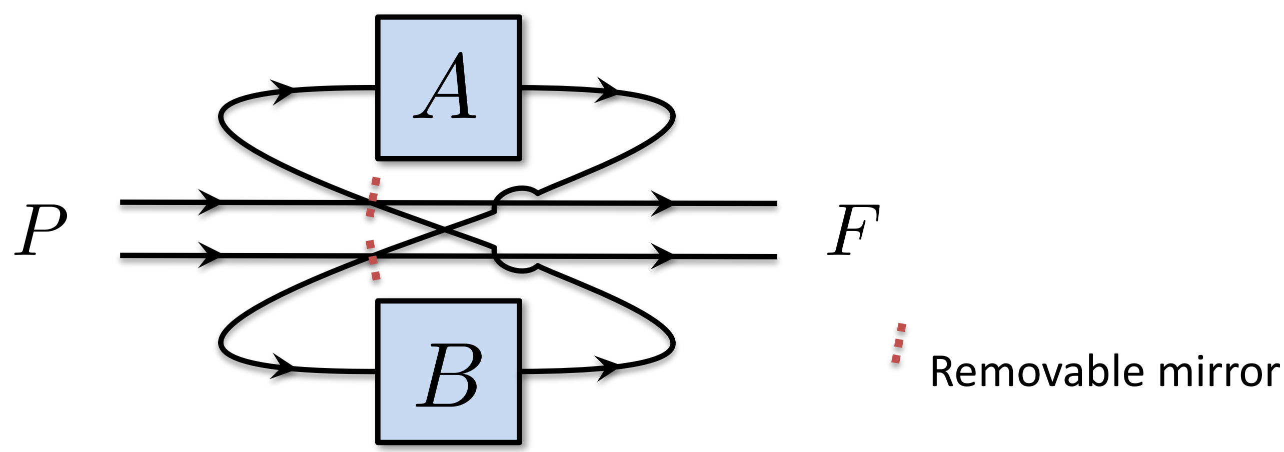
The Quantum switch

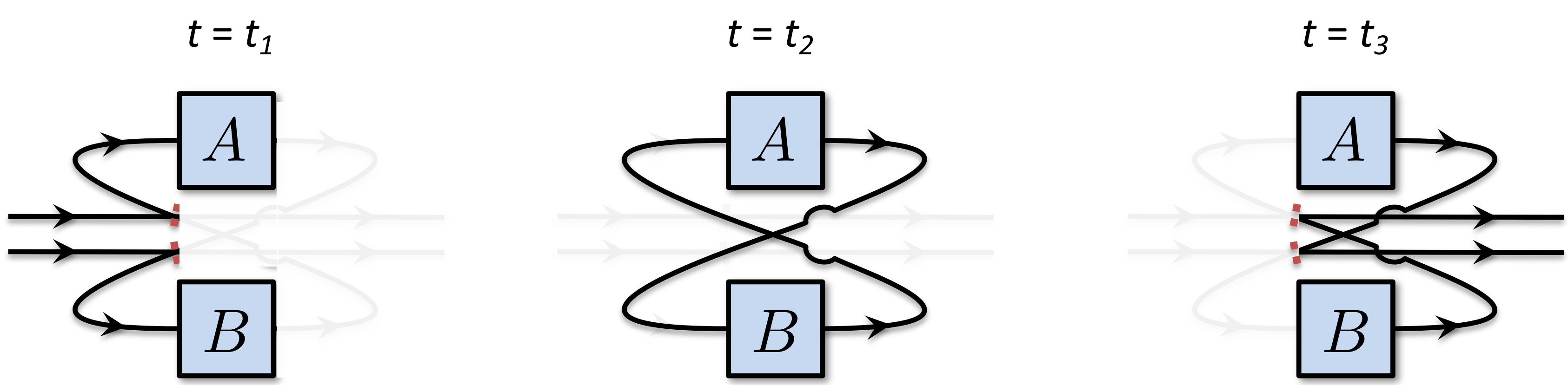
- Another possible implementation:



The Quantum switch

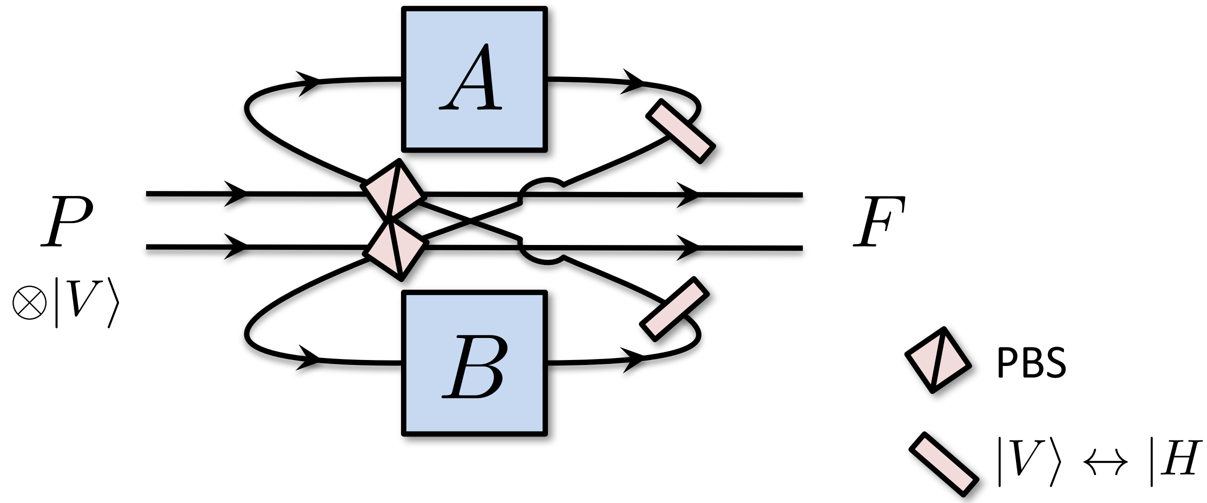
- Another possible implementation:



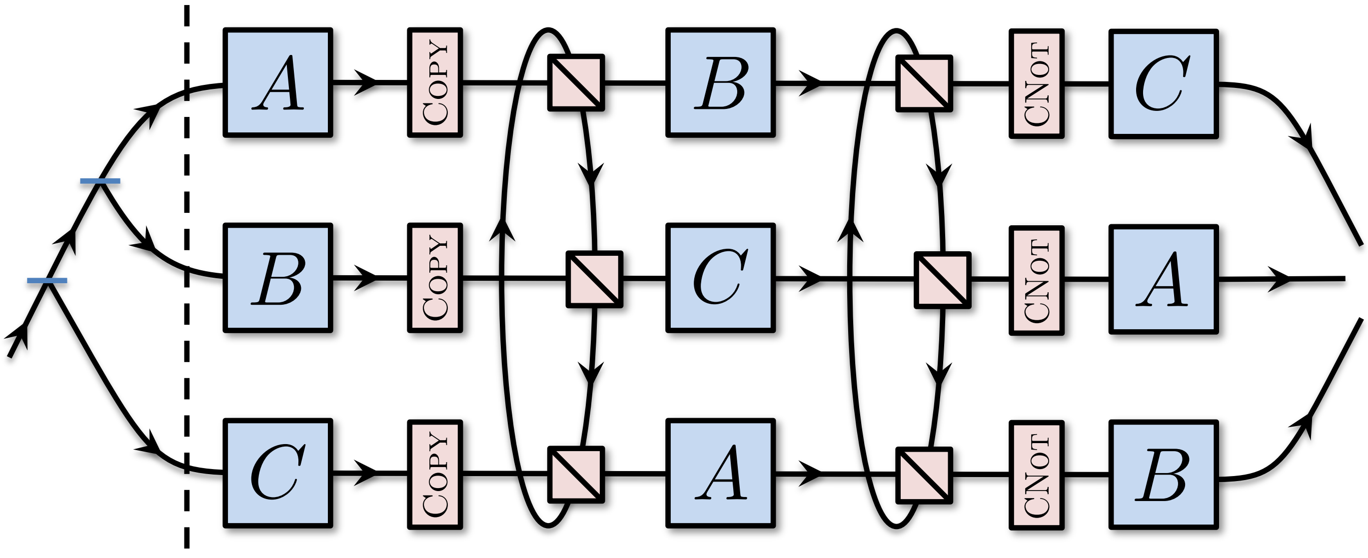


The Quantum switch

- Another possible implementation:

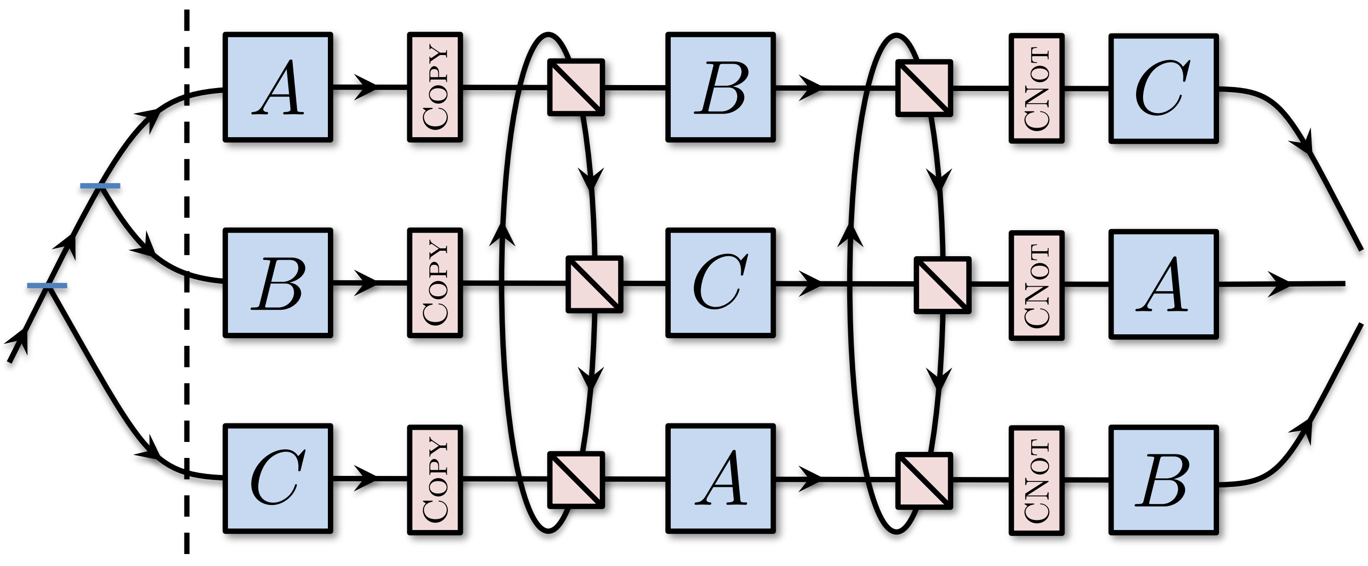


Some new QC-QC

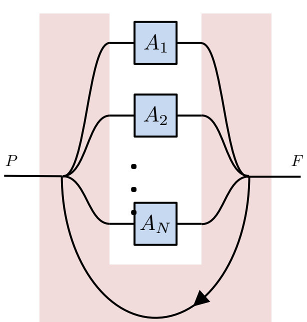


$|\psi\rangle^t \otimes \frac{1}{\sqrt{3}} \sum_{k_1} |k_1\rangle^{C_1^{\text{path}}}$

Some new QC-QC

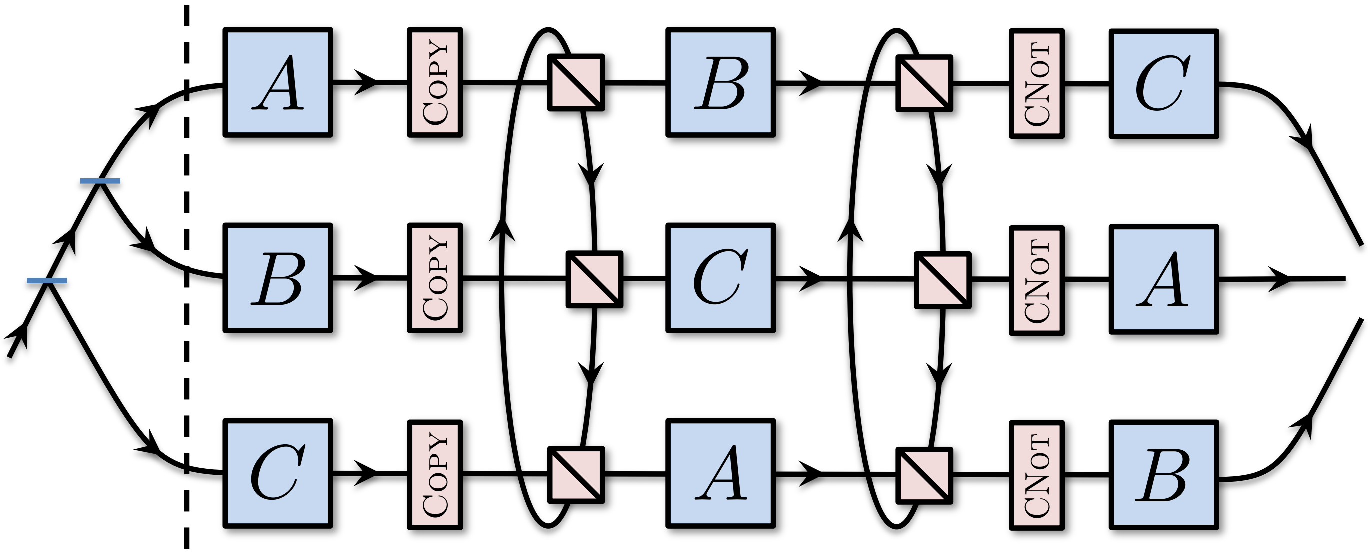


$$|\psi\rangle^t \otimes \frac{1}{\sqrt{3}} \sum_{k_1} |k_1\rangle^{C_1^{\text{path}}}$$



(Should loop back…)

Some new QC-QC



$|\psi\rangle^t \otimes \frac{1}{\sqrt{3}} \sum_{k_1} |k_1\rangle^{C_1^{\mathrm{path}}}$

- Causally nonseparable
- Exhibits both dynamical & coherently-controlled causal order

Possible applications…

- Quantum circuits = supermaps = "higher-order" operations
  - Applications for "higher-order" quantum info tasks
    (to calculate functions of maps)

Possible applications…

- Quantum circuits = supermaps = "higher-order" operations
  - Applications for "higher-order" quantum info tasks
    (to calculate functions of maps)

e.g.:

- given 2 black-box operations *A* and *B*:
  do *A* and *B* commute or anti-commute? [Chiribella, PRA 2012]
- cloning "black-box" gates
- inverting / transposing a black-box operation [→ cf Marco Tulio Quintino's talk]
- …

Possible applications…

- Quantum circuits = supermaps = "higher-order" operations
  - Applications for "higher-order" quantum info tasks
    (to calculate functions of maps)

e.g.:

- Given U_1, U_2, U_i as black boxes,
  determine whether $U_i = U_1$ or $U_i = U_2$

[Shimbo, Soeda, Murao, arXiv 2018]

Possible applications…

- Quantum circuits = supermaps = "higher-order" operations
  - Applications for "higher-order" quantum info tasks (to calculate functions of maps)

e.g.:

- Given U_1, U_2, U_i as black boxes, determine whether $U_i = U_1$ or $U_i = U_2$

[Shimbo, Soeda, Murao, arXiv 2018]

(qubits, Haar-random U's)

- P^{succ}(Parallel) = P^{succ}(QC-FO) = P^{succ}(QC-CC) = P^{succ}(QC-QC) = P^{succ}(gen. W)

Possible applications…

- Quantum circuits = supermaps = "higher-order" operations
  - Applications for "higher-order" quantum info tasks (to calculate functions of maps)

e.g.:

- Given U_1, U_2, U_i as black boxes, determine whether $U_i = U_1$ or $U_i = U_2$

[Shimbo, Soeda, Murao, arXiv 2018]

(qubits, Haar-random U's)
- P^{succ}(Parallel) = P^{succ}(QC-FO) = P^{succ}(QC-CC) = P^{succ}(QC-QC) = P^{succ}(gen. W)

- Given U_1, U_2, U_3, U_i as black boxes, determine whether $U_i = U_1$ or $U_i = U_2$ or $U_i = U_3$

(qubits, Haar-random U's)
- P^{succ}(Parallel) < P^{succ}(QC-FO) ≈ P^{succ}(QC-CC) < P^{succ}(QC-QC) < P^{succ}(gen. W)

Violations of causal inequalities with QC-QCs?

Violations of causal inequalities with QC-QCs?

No.

W is a QC-QC iff $\exists \{W_{(\mathcal{K},\ell)} \geq 0\}$,

$$\mathrm{Tr}_F W = \sum_{k\in\mathcal{N}} W_{(\mathcal{N}\backslash\{k\},k)} \otimes \mathbb{1}^{A_k^O},$$

$$\forall\, \emptyset \subsetneq \mathcal{K} \subsetneq \mathcal{N},\ \sum_{\ell\in\mathcal{N}\backslash\mathcal{K}} \mathrm{Tr}_{A_\ell^I} W_{(\mathcal{K},\ell)} = \sum_{k\in\mathcal{K}} W_{(\mathcal{K}\backslash\{k\},k)} \otimes \mathbb{1}^{A_k^O},$$

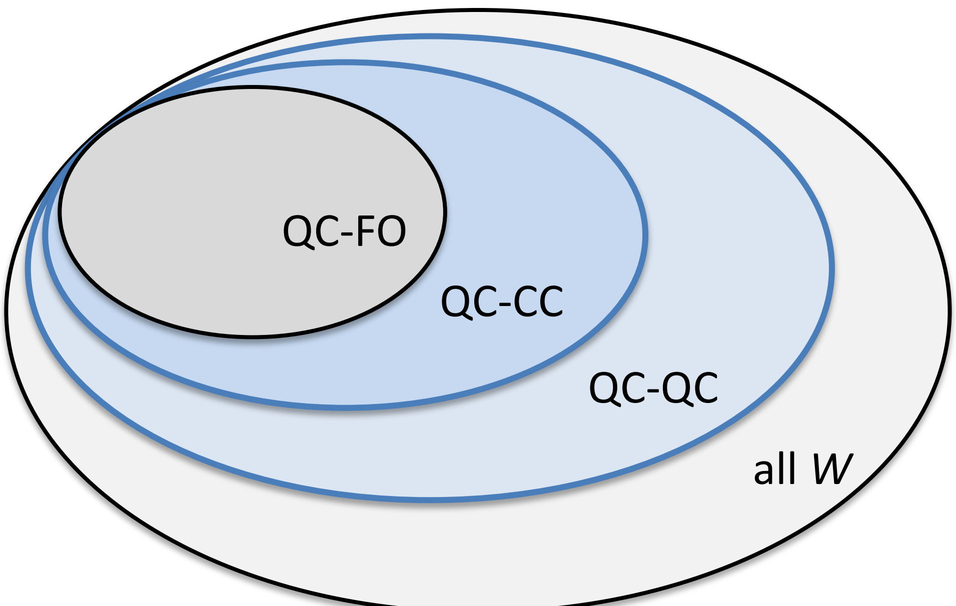
$$\text{and} \quad \sum_{\ell\in\mathcal{N}} \mathrm{Tr}_{A_\ell^I} W_{(\emptyset,\ell)} = \mathbb{1}^P.$$

Imposes some structure to the correlations obtained from *W:*

these are “causal”

Conclusion

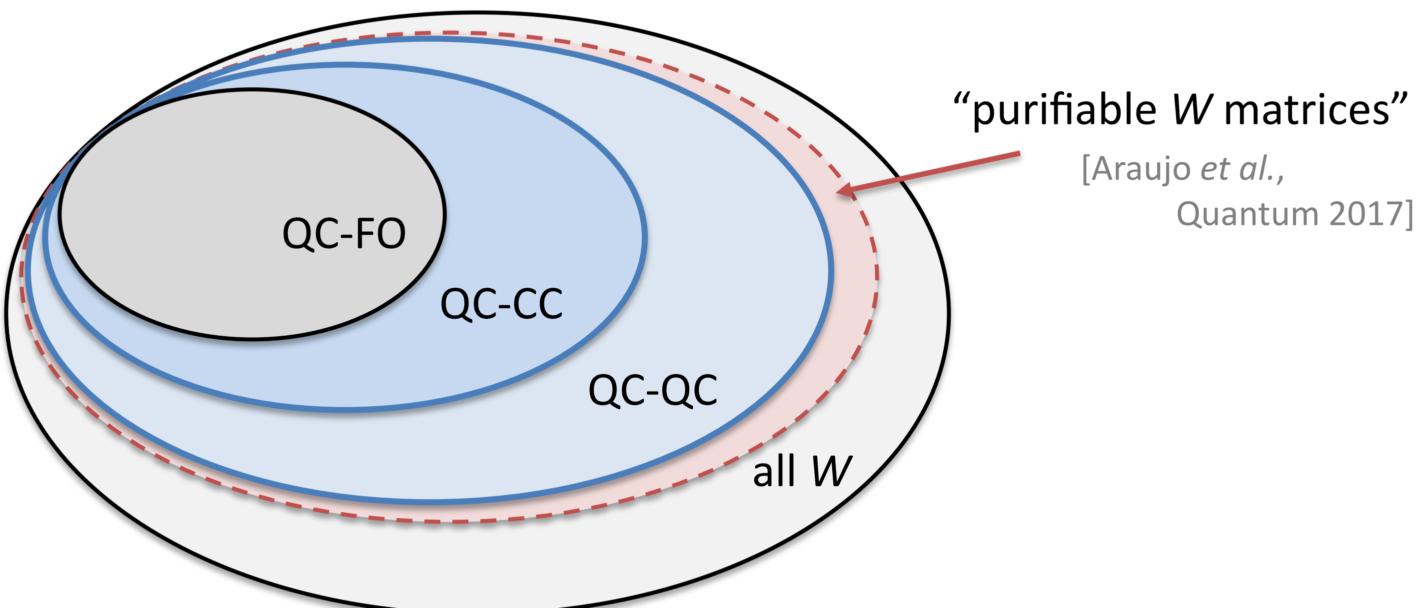
- Quantum circuits as “quantum supermaps”: description & characterisation (via SDP) of
  - ➢ Quantum circuits w/ fixed causal order
  - ➢ Quantum circuits w/ classical control of causal order (-> causally sep. processes)
  - ➢ Quantum circuits w/ quantum control of causal order (-> can be causally nonsep.)



[Wechs *et al.*, in prep.]

Conclusion

- Quantum circuits as "quantum supermaps": description & characterisation (via SDP) of
  - ➢ Quantum circuits w/ fixed causal order
  - ➢ Quantum circuits w/ classical control of causal order (-> causally sep. processes)
  - ➢ Quantum circuits w/ quantum control of causal order (-> can be causally nonsep.)



[Wechs *et al.*, in prep.]

Conclusion

- Quantum circuits as "quantum supermaps": description & characterisation (via SDP) of
  - ➢ Quantum circuits w/ fixed causal order
  - ➢ Quantum circuits w/ classical control of causal order (-> causally sep. processes)
  - ➢ Quantum circuits w/ quantum control of causal order (-> can be causally nonsep.)

- **New types of quantum processes w/ indefinite causal order**,
  beyond the quantum switch: there's **more to play with!** ☺
  - ➢ Towards new applications for QIP

[Wechs *et al.*, in prep.]

Conclusion

- Quantum circuits as "quantum supermaps": description & characterisation (via SDP) of
  - ➢ Quantum circuits w/ fixed causal order
  - ➢ Quantum circuits w/ classical control of causal order (-> causally sep. processes)
  - ➢ Quantum circuits w/ quantum control of causal order (-> can be causally nonsep.)

- **New types of quantum processes w/ indefinite causal order**, beyond the quantum switch: there's **more to play with!** ☺
  - ➢ Towards new applications for QIP

- "**Gravitational** QC-QCs"?

Conclusion

- Quantum circuits as "quantum supermaps": description & characterisation (via SDP) of
  - ➢ Quantum circuits w/ fixed causal order
  - ➢ Quantum circuits w/ classical control of causal order (-> causally sep. processes)
  - ➢ Quantum circuits w/ quantum control of causal order (-> can be causally nonsep.)

- **New types of quantum processes w/ indefinite causal order**, beyond the quantum switch: there's **more to play with!** ☺
  - ➢ Towards new applications for QIP

- "**Gravitational** QC-QCs"?

- More general process matrices w/ quantum realisation?
  Causal inequality-violating processes??

Thank you for your attention

Joint work with

Julian Wechs

Hippolyte Dourdent

Alastair Abbott

Quantum Engineering
Univ. Grenoble Alpes

UGA
Univ. Grenoble Alpes

ANR

Violations of causal inequalities with QC-QCs?

No.

$$W \text{ is a QC-QC iff } \exists\{W_{(\mathcal{K},\ell)} \geq 0\},$$

$$\mathrm{Tr}_F W = \sum_{k\in\mathcal{N}} W_{(\mathcal{N}\backslash\{k\},k)} \otimes \mathbb{1}^{A_k^O},$$

$$\forall \emptyset \subsetneq \mathcal{K} \subsetneq \mathcal{N}, \sum_{\ell\in\mathcal{N}\backslash\mathcal{K}} \mathrm{Tr}_{A_\ell^I} W_{(\mathcal{K},\ell)} = \sum_{k\in\mathcal{K}} W_{(\mathcal{K}\backslash\{k\},k)} \otimes \mathbb{1}^{A_k^O},$$

$$\text{and} \quad \sum_{\ell\in\mathcal{N}} \mathrm{Tr}_{A_\ell^I} W_{(\emptyset,\ell)} = \mathbb{1}^P.$$

Defining

$$r_{(\mathcal{K},\ell)}(\vec{a}_\mathcal{K}|\vec{x}_\mathcal{K}) = \mathrm{Tr}\big[\big(\mathrm{Tr}_{A_\ell^I} W_{(\mathcal{K},\ell)}\big)\, M^T_{\vec{a}_\mathcal{K}|\vec{x}_\mathcal{K}}\big],$$

$$s_{(\mathcal{K},\ell)}(\vec{a}_{\mathcal{K},\ell}|\vec{x}_{\mathcal{K},\ell}) = \mathrm{Tr}\big[\big(W_{(\mathcal{K},\ell)} \otimes \mathbb{1}^{A_\ell^O}\big) M^T_{\vec{a}_{\mathcal{K},\ell}|\vec{x}_{\mathcal{K},\ell}}\big],$$

we have

$$\sum_{\ell\in\mathcal{N}\backslash\mathcal{K}} r_{(\mathcal{K},\ell)}(\vec{a}_\mathcal{K}|\vec{x}_\mathcal{K}) = \sum_{k\in\mathcal{K}} s_{(\mathcal{K}\backslash\{k\},k)}(\vec{a}_\mathcal{K}|\vec{x}_\mathcal{K}) \quad := q_\mathcal{K}(\vec{a}_\mathcal{K}|\vec{x}_\mathcal{K})$$

Can show recursively that

$$\forall n, P(\vec{a}|\vec{x}) = \sum_{|\mathcal{K}|=n-1} \underbrace{q_\mathcal{K}(\vec{a}_\mathcal{K}|\vec{x}_\mathcal{K})}_{1 \text{ for } |\mathcal{K}|=0} P^{causal}_{\mathcal{K},\vec{x}_\mathcal{K},\vec{a}_\mathcal{K}}(\vec{a}_{\mathcal{N}\backslash\mathcal{K}}|\vec{x}_{\mathcal{N}\backslash\mathcal{K}})$$